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Reliability modeling based on Maximum Entropy and non-central moments as an alternative for RCM schemes or replaceable systems

Modelización de la fiabilidad basada en la máxima entropía y los momentos no centrales como alternativa para los esquemas RCM o los sistemas reemplazables

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Abstract

This paper offers an alternative to determine reliability-centered maintenance (RCM) schemes for replaceable systems, when replacement times are censored and only the information that maintenance technicians, from the subjectivity of their experience, is available. Using differential entropy in information theory, and exploiting Lagrangian optimization algorithms, a Generalized Probability Density of Maximum Entropy (GPDME) is extracted. Lagrangian techniques provide a set of parameters that characterize the GPDME, the estimation of the parameters is done by first order perturbation of the integral of non-central moments, with which, the GPDME is typically built. In the emerging industry, RCM maintenance plans are not a common standard, in an attempt to put into practice, the benefits of RCM to this industrial segment, a case study, where the presented methodology was applied is provided. In the discussion and conclusions section, the areas of opportunity that are observed in the methodology presented in this work are adressed.

RCM-Reliability, Entropy, Lagrangian

Resumen

Este trabajo ofrece una alternativa para determinar esquemas de mantenimiento centrados en la fiabilidad (RCM) para sistemas reemplazables, cuando los tiempos de sustitución están censurados y sólo se dispone de la información que los técnicos de mantenimiento, a partir de la subjetividad de su experiencia. Utilizando la entropía diferencial de la teoría de la información, y explotando los algoritmos de optimización lagrangianos, se extrae una Densidad de Probabilidad Generalizada de Máxima Entropía (GPDME). Las técnicas lagrangianas proporcionan un conjunto de parámetros que caracterizan la GPDME, la estimación de los parámetros se realiza mediante la perturbación de primer orden de la integral de momentos no centrales, con la que, típicamente, se construye la GPDME. En la industria emergente, los planes de mantenimiento RCM no son un estándar común, en un intento de poner en práctica, los beneficios de RCM a este segmento industrial, se proporciona un caso de estudio, donde se aplicó la metodología presentada. En la sección de discusión y conclusiones, se abordan las áreas de oportunidad que se observan en la metodología presentada en este trabajo.

RCM-Fiabilidad, Entropía, Lagrangiano

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Introduction

In the reliability-centred maintenance (RCM) framework, it is common to have difficulties regarding the failure frequency of vital components used in industry. Usually maintenance records [2], information provided by manufacturers in the marketing of these components [3] and the experience of maintenance personnel are the primary sources for creating a maintenance plan.

In practice, most of this data is either non-existent [1], relies on subjective views or is censored in the sense that the components have not failed. At the same time, maintenance plans represent an investment in fixed assets, personnel training and spare parts supply that sometimes exceeds what medium-sized companies can afford in the short term.

It is in this segment that the need for reliability-based maintenance plans is detected, working primarily with very small samples, only censored data or personnel experience. With this information and the concepts of entropy in information theory, a generalized model is developed that describes the probability of failure of a severely reduced sample of failure data in industrial components.

1. Problem Presentation

One of the most widely used probability densities to study the failure frequency in the RCM framework is the Weibull [10], [13], the scale parameter is determined by the sample size, if the sample is very small, the Weibull probability density is still able to give reliable results on a reduced sample.

On the other hand, if there is no failure data, the data is censored [13], therefore, the estimator for the Weibull probability density cannot be determined:

$$\hat{\eta} = \left(\frac{1}{r}\sum_{i=1}^{n} t_i^{\beta}\right)^{1/\beta} \quad (1)$$

In (1), r is the number of data, being censored, r=0, which indeterminates the scale parameter.

2. General Objective

To build a model that can estimate the probability of failure in industrial components from severely reduced samples or censored data samples, that is, in those replaceable components in which the failure has not yet occurred and the maintenance records are unreliable or non-existent and we only have the experience of the maintenance personnel and the manufacturer's data. [2] [3] [4] [17]

3. Theoretical considerations.

3.1. Reliability and the Weibull probability density

If x is a Continuous Random Variable (C.R.V.) that records the time elapsed before having a failure in some replaceable artefact, η and β, non-zero positive real constants, we define η as scale parameter, and β as shape parameter, the Weibull probability distribution is given by:

$$f(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} e^{-\left(\frac{x}{\eta}\right)^{\beta}}; x \in [0, \infty] \quad (2)$$

The literature related to sampling densities is abundant, however, in RCM [13], the Weibull represents a viable first option due to its analytical benefits when establishing it as a reliability model. Reliability is defined as:

$$R(T) = 1 - F(x \leq T) \quad (3)$$

Where T is the Mean Time Before Failure (MTTF) and F(x≤T) is the cumulative probability function. It can be seen trivially that (1) and (2) lead to:

$$R(T) = e^{-\left(\frac{x}{\eta}\right)^{\beta}} \quad (4)$$

Once reliability is available, the rest of the relevant functions for determining a reliability-centred maintenance scheme follow directly, as the MTTF in reliability is defined by means of:

$$E(t) = \int_0^{\infty} R(t)dt \quad (5)$$

However, as determined by equation (1), if the data are censored (which is to say that they are service data, not failure data), a reliability-centred maintenance scheme cannot be fundamentally established [13].

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In the following, an independent approach is developed, based on classical results from information theory, C. Shannon's differential entropy, functional analysis and probability in terms of Kolmogorov's axioms to obtain a generalized probability density that does not depend on failure data and can be interpreted in service data schemes.

3.2. Fundamentals of information theory

Entropy S quantifies the lack of information of a random experiment. If the experiment is governed by a probability distribution p_i then the function measuring the uncertainty is defined as:

$$S = -k \sum_{i=1}^{n} p_i \ln(p_i) \qquad (6)$$

and it is assumed that 0⩽p_i ⩽1 which ensures the monotonicity property of entropy: S⩾0 over the whole path of p_i, from a linear point of view, S represents a basis that diagonalize pˆ[12]. Thus, when we possess the maximum information of the system, S reaches its minimum value. In this idea, S will reach its maximum value when all p_i are equiprobable. Following this hypothesis, we could assign probabilities by means of the relative frequency p_i=1/n which directly associates the entropy: S=kln(n), so that S grows in direct proportion to n in the mentioned case. In general, [5] [2] [14] the fundamental hypothesis of this work is based on the assumption that the entropy satisfies the following

Definition: *Maximum entropy principle:*

"The statistical entropy of a random system reaches the maximum compatible with the imposed constraints."

The basic idea behind this principle is that there is no reason to privilege a particular state or event in a random experiment. Starting from a state of equiprobability, there is uniquely a probability density defined over the states accessible to entropy from the point of view of information theory.

3.4. Optimisation and variational techniques. Euler-Lagrange equations.

If one has a well-defined functional in some region of R^2, by considering the action on the functional defined with the re-parameterisation: Y(x)=y(x)+εα(x) and varying the action with respect to the parameter ε, given that α(x) is an arbitrary function within the defined domain, one obtains the Euler equation:

$$\frac{d}{dx}\frac{\partial F}{\partial y'} - \frac{\partial F}{\partial x} = 0 \qquad (7)$$

By exploiting the properties of (7) in terms of an isoperimetric optimization problem it is possible to make an identification with the Lagrange multiplier optimization method [14], to extract a maximum entropy probability density.

3.5. Generalised probability density

Equation (7) can be used to define a functional when subjected to continuity conditions on its assumed random variable. In this sense, the Shanon entropy [5] [6] [17] can be defined as:

$$S(P, P') = -\int f(t) \ln[f(t)] dt \qquad (8)$$

Where f(t) is a probability density defined in equation (6) for the continuous case, and the integration extends over the entire domain of f(t).Subjecting (8) to the constraints:

$$\int f(t) dt = 1$$

$$\int t^k f(t) dt = \mu_k \qquad (9)$$

Where μ_k is the k-th moment of f. Equations (8) and (9) can be stated as an optimization problem by Lagrange multipliers, the functional (or objective function) is constructed directly:

$$F(f, f', t) = -f(t) \ln[f(t)] - \sum_{i=1}^{n} \lambda_k t^k f(t) \quad (10)$$

Where the λ_k are the Lagrange multipliers. Applying equation (7) to equation (10) gives:

$$f(t) = Ae^{-\varphi(t)} \qquad (11)$$

Where A is a normalisation constant and

$$\varphi(t) = 1 + \sum_{i=1}^{n} \lambda_k t^k f(t) \qquad (12)$$

To ensure convergence of the probability model, we need to require that:

N=2m with m a [17] positive integer.

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3.7. Lagrange multipliers

We can obtain a set of equations from which to extract the Lagrange multipliers [9] [14]. One way to do this is to assume that φ(t) forms a basis of orthogonal functions, in which case, taking the derivative of (11), integrating over its entire domain and applying boundary conditions on the maximum entropy probability density, we get:

$$\int_{-\infty}^{\infty} \frac{d}{dt} f dt = 0 \tag{13}$$

In other words:

$$\lambda_k k \sum_{k=1}^{N} \int_{-\infty}^{\infty} t^{k-1} f(t) dt = 0 \tag{14}$$

In the above formulation, one has a series of moments defined for each integer value of k. Likewise, the argument used leading to (14) can be extended if it is derived to the probability model, multiplied by t^n and integrated over its entire domain, the new parameter n will account for the constraints that can be imposed on the functional arising from the Euler-Lagrange equation, proceeding, one obtains:

$$\int_{-\infty}^{\infty} t^n \frac{d}{dx} f dt = \lambda_k k \sum_{k=1}^{N} \int_{-\infty}^{\infty} t^{k+n-1} f dt \tag{15}$$

It is important to note that (14) and (15) are series of moments when written explicitly, they are respectively:

$$\lambda_1 + 2\lambda_2\mu_1 + 3\lambda_3\mu_2 + \cdots + N\lambda_N\mu_{N-1} = 0$$

$$\lambda_1\mu_n + 2\lambda_2\mu_{n+1} \ldots + N\lambda_N\mu_{n+N-1} = n\mu_{n-1}$$

Therefore, (14) and (15) form the core of the proposal presented in this article, from the theoretical perspective, [8] [9] [14] [17] since, in reality, (14) and (15) form a system of equations given by:

$$\begin{pmatrix} \mu_1 & \cdots & \mu_{N-1} \\ \vdots & \ddots & \vdots \\ \mu_{N+1} & \cdots & \mu_{2N} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \vdots \\ N\lambda_N \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2\mu_2 \\ \vdots \\ (N+1)\mu_N \end{pmatrix} \tag{16}$$

In equation (16) it is then necessary to feed the moments operator with some reasonable estimate of the moments, and in such a case, proceed to solve the system of equations for the Lagrange multipliers, which are the parameters of the generalized probability model given in (11).

The estimation of the moments of a probability density can be carried out through several alternatives, in this work the approach is based on estimating the moments by means of a non-central moment integral [12] [14], assuming that the generalized maximum entropy probability model does not deviate much from that obtained by means of a Weibull probability density. That is, it is assumed that there is a minimal perturbation [6], between the Weibull model and the Generalized probability model.

4. RCM methodology

The usual reliability-centred maintenance schemes require identifying the characteristics and processes to which various components are subjected in various stages, with the aim of selecting the appropriate maintenance scheme that responds to the industrial process in which a component or a set of components that can be replaceable or repairable is working. Roughly speaking, the 6 usual stages of reliability-focused maintenance are: [13].

1. Equipment selection
2. Fault identification
3. Define fault diagrams. Identify consequences.
4. Maintenance strategy
5. Reliability
6. Reliability focused maintenance plan.

In this paper, the focus is on the fifth stage. Reliability in the RCM scheme consists of selecting the best model that represents the probability of failure and building with it the whole reliability-centred maintenance scheme. In other words, the starting point is to select a specific component, identify its failure modes, understand the consequences of the failure of the selected equipment or component and propose a maintenance strategy based on the modelling of a probability density.

Established in the fifth step above, the question arises, Is the sample large enough?

In the following section we present an example where the scheme described in this paper was applied, the sample obtained was reinterpreted for the specific purposes of the case study and the development of the methodologies outlined in this section.

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5. Case Study

The following data were obtained from the company Plásticos y Metales De Coahuila, and are the operating times in hours of valves used to control the pressure with which plastic is injected into various moulds. They represent an example of a system of replaceable parts and can be analysed within the schemes of reliability-centred maintenance.

23880
24210
27212
25731
25687
28213
25231
21782
33121
25103

Table 1

The technician's experience indicates that the valves will fail between 23200 and 27900 operating hours.

5.1. Weibull analysis

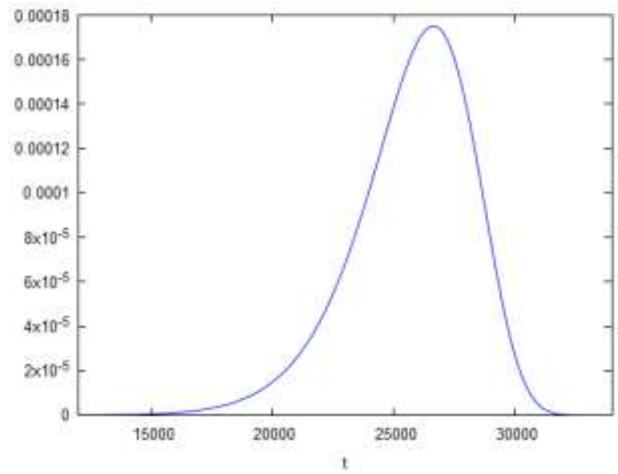
In the following it is assumed that the data provided is a sample of failure times. Also, all graphs provided from this point onwards are plotted in hours on the abscissa axis.

In order to provide a reliability-centred maintenance scheme, by carrying out a Weibull parameter adjustment [10] using the maximum likelihood method. With the scale parameter at the mean of the data and the shape parameter equal to one, this initial data provides through the Newton-Rhapson method the values:

η=26818.4; β=12.7290

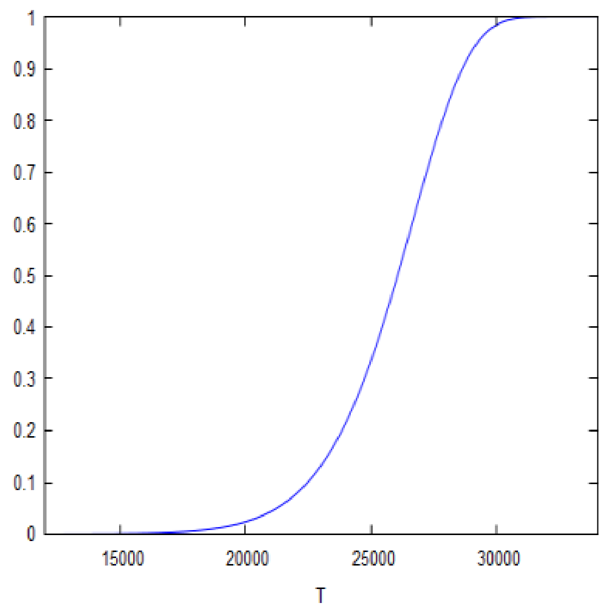
The Weibull probability density that with maximum likelihood represents the data in Table 1 is thus:

$$W(t) = 5.436 \times 10^{-59} t^{11.729} e^{-4.27\times10^{-57} t^{12.73}}$$



Graph 1

The cumulative probability function is then:



Graph 2

5.2. Mathematical expectation, MTTF

We then proceed to determine the MTTF, since we have the reliability function, the MTTF turns out to be a value of critical importance for making decisions regarding the maintenance scheme focused on reliability, (5) provides it:

$$E(T) = 25676.21 \quad (17)$$

Likewise, the instantaneous failure frequency is given by:

$$h(t) = \frac{d}{dt} ln(R(t)) \quad (18)$$

The instantaneous failure frequency, also known as the risk function, indicates the rate at which reliability decays as the time the device in question is in operation evolves. The risk function, like reliability, decreases abruptly after the MTTF, inducing a risk process if at least inspection measures are not taken at around 26000 hours of valve operation. It is common practice not to wait for the valve to fail, but to replace it before any malfunction occurs. However, in Table 1 the data is interpreted as failure data, not service times. [1]

5.3. Reliability-centred maintenance scheme. RCM

With the data provided by the company Plásticos y Metales de Coahuila, the following reliability-centred maintenance scheme is established.

Weibull model	parameters	MTTF
$R(T) = e^{-\left(\frac{x}{\eta}\right)^{\beta}}$	η=26818.4	~25700hrs
	β=12.7290	h(t)>26000hrs

Table 2

Thus, it would be recommended to the company's management to replace the valves before 25000 hours of operation without waiting to reach the MTTF, as the risk of failure increases abruptly around 25700 hours.

Furthermore, if we consider the value of the probability of failure at 28000 hours, we can see that it represents a high risk for the machinery:

F (28000) =0.82293

5.4. Reinterpreting the sample

In this section we reinterpret the data to compare it with the results of the previous section. It is necessary to understand that what is being offered is an example of how to work with the maximum entropy model when the data are censored and only the expertise of the technician is available. In no way does the following scheme correspond to a complete approach on how to elaborate the estimation of the moments in equation (16), it is intended, however, to give an indicative idea of how to proceed once a complete theoretical scheme for the estimation of moments is available.

In fact, the status and progress of the presented research is formally shown up to the momentum operator and the theoretical framework supporting the maximum entropy probability density. However, it is interesting to note that the scheme proposed through equation (16) to determine the Lagrange multipliers has remarkable utility even when only reasonable estimates of the first moments of the maximum entropy probability density are known.

Since these are replacement times, no actual failure data is available and therefore the entire sample is censored, only the technician's experience is available. The usual RCM schemes suggest the use of a Weibull probability density, but when trying to run the estimation of the Weibull parameters, the problem already anticipated in the introduction is encountered.

The alternative proposed is a probability density that comes from maximize entropy, i.e., a model identical to the one proposed in equation (11) is considered, and it is assumed that the Weibull probability density of service data can be used as an initial perturbation point to construct the probability density of maximum entropy.

Such a choice in the model and equation (16) leads to consider some system of equations, previously estimating the moments of the function f(t), and of course, the question arises, how many terms to include in φ(t) to adequately represent the model? If one decides to include the mean, variance, skewness and kurtosis, the operator of moments is 4X4, however [9] [10] [11] [15] suggest considering the first four moments, even pointing out that when the information in the operator of moments is extremely difficult to obtain in the context of achieving a basis, which diagonalises the operator of moments, the first two moments are sufficient to determine in an acceptable way the probability density. In such a case, will have a system of 2X2 equations, which is much more computationally friendly to deal with.

5.5. Estimation of moments

To estimate the moments of the censored data [4], the MTTF obtained from the Weibull analysis will be used assuming that the mean and variance parameters do not deviate much from those predicted by the Weibull analysis [10] [12].

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The technician's experience indicates that the service times are made in the interval (24000,28000) in hours, from which the following values are obtained for the moment operator:

μ_1 =26818.4; μ_2 = 6079355.823

Since no information is available, the variance has been estimated by the non-central moments formula, using the Weibull probability density [13][14][15], previously determined to locate the maximum entropy probability density:

$$\mu_2 = \int_0^{\infty}(t-\mu_1)^2 W[\eta,\beta]dt + \sum_{k=1}^{N}|\phi(\mu_i)| \quad (19)$$

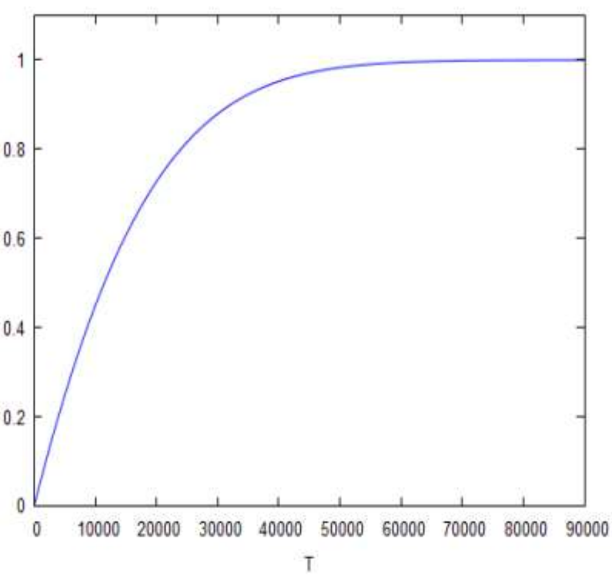
The rest of the non-central moments, [4] [6] form a convergent residue such that $|\phi(\mu_i)|^2 \rightarrow 0$ as we introduce more moments in the maximum entropy probability density [12] [16] [17]. Solving (16) with the estimated moments yields the model:

$$f(t) = A\mathrm{e}^{-(1+3.56901\times10^{-5}t+6.4271\times10^{-10}t^2)} \quad (20)$$

(20) is already a maximum entropy probability density. The normalization constant takes the value:

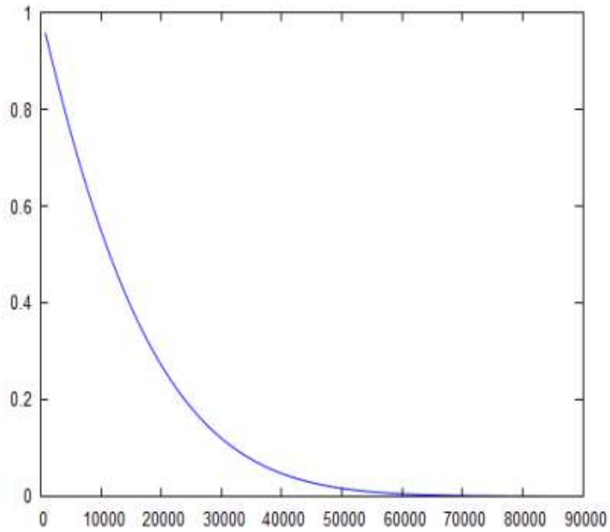
$A = 1/6741.17713646257$

whereby the non-parametric maximum entropy probability model has the graph:



Graph 4

Likewise, the reliability function has the following graphs



Graph 3

The MTTF obtainded from equation (5), for this distribution is therefore:

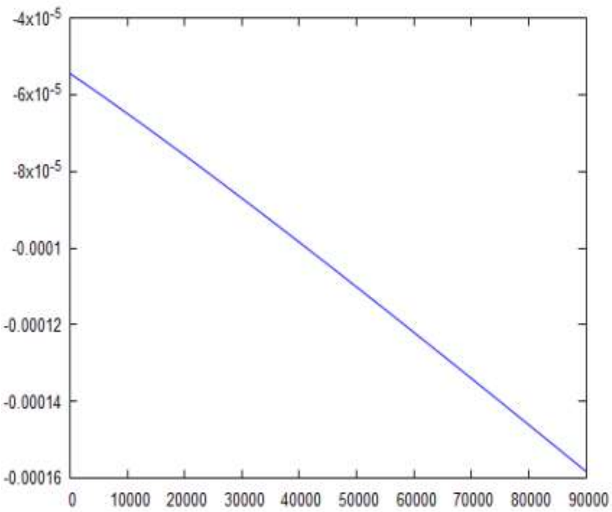
$E(T)$ =14600

Thus, the MTTF indicates a situation of lack of maintenance, putting the components of the entire injection system at risk. In the same way as was done with the Weibull probability density, when evaluating the probability of failure in 28000 hours, the following is obtained:

$$F(28000) = 0.85746 \quad (21)$$

With both models, about 84% of the times when valves are allowed to operate above 28000 hours, the probability of failure is very high. Thus, the conclusions of extreme operating times obtained with the Weibull can also be obtained with the maximum entropy model.

However, having used only two moments in equation (16) it is possible that the model does not adequately represent the information provided, i.e. replacement times and technician experience. One way to indirectly see if the calculated probability actually represents the phenomenon being treated is provided by the hazard function [12] [13], equation (18), which indicates the rate at which reliability decreases as time evolves. It is in itself a way to predict how the reliability decreases as long as the components keep working, given that we have R(T), we can determine h(t) and observe if the rate at which it decreases around the MTTF is too steep, the graph of h(t) associated to the reliability calculated by maximum entropy is:



Graph 5

It can be seen that at around 15500 hours of valve operation, the risk continues to decrease, practically at the same rate. This means that the proposed model can be improved by virtue of the risk function: "it is still possible that there is another model that better represents the information available".

6. Discussion and conclusions

The state of the research indicates that it is possible to elaborate reliability-centred maintenance plans with the tool developed up to this point, further developments will strengthen the proposed schemes and will give much more certainty to the maintenance plans to be elaborated for the indicated industrial sector. However, the need for a more formal scheme to estimate the moments [15] [16] [18] that feed equation (16) is missing, the research line contains parallel work in fuzzy logic and stochastic parameter location algorithms, based on the convolution theorem [14].

On the other hand, given that the formal language of probability is measure theory, one cannot rule out venturing into this field to extract the necessary tools to allow for a significant improvement of the results and future research planned in the present line of research.

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