Impact of COVID-19 on Fractal Capital Market Recursion

Impacto del COVID-19 en la recursividad fractal del mercado de capitales

RAMOS-ESCAMILLA, María†*

ECORFAN-Mexico, S.C.


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Abstract

We present the optimization of the current prices function of the Mexican Stock Exchange index and we will focus our attention on aspects of maximizing stock market margins with limits in the bookkeeping operation. We ourselves will model the possible stochastic recursion scenarios in "n" as a fractal annihilation factor within this stock market. Finally we will obtain the expected delta of the price range and its Hamiltonian to minimize the operational risk of the capital market with the presence of COVID-19 in Mexico.


Resumen

Presentamos la optimización de la función de precios corrientes del índice de la Bolsa de México y enfocaremos la atención en aspectos de maximización de márgenes bursátiles con cotas de límites en la operación de teneduría. A sí mismo modelaremos los posibles escenarios de recursividad estocástica en “n” como factor de aniquilación fractal dentro de esta bolsa de valores. Finalmente obtendremos la delta esperada del rango de precios y su hamiltoniano para minimizar el riesgo operacional del mercado de capitales con presencia del COVID-19 en México.

Abstract

Fractal, COVID-19, Prices

Fractal, COVID-19, Precios

* Correspondence to Author (email: ramos@ecorfan.org)
† Researcher contributing first author.
Introduction

The companies of the BMV are capitalized and their instruments tend to rise in price. Let's start with defining the discrete dynamics system as a pair \((x, f)\) where \(x\) is a field and \(f : X \to X\). Given a point \(x \in X\), then \(\{x, f(x), f^2(x), f^3(x), \ldots\}\) will be called the orbit of, where \(f^n(x) = f \circ \cdots \circ f(x)\), therefore we consider the classification of the fixed points according to their properties in a complex dynamic system \((C, f)\) [Mandelbrot, M: 1978], are the following:

- \(z_0 \in C\), \(z_0\) it is an attractor point if \(|f'(z_0)| < 1\)
- \(z_0 \in C\), \(z_0\) it is a repulsive point if \(|f'(z_0)| > 1\)
- \(z_0 \in C\), \(z_0\) is an indifferent point if \(|f'(z_0)| = 1\)
- \(z_0 \in C\), \(z_0\) it is a super attractor point if \(|f'(z_0)| = 0\)

The dimension tells us how many additional pieces of an object can be revealed as the resolution is more precise and there are three different ways to evaluate it: the fractal dimension, the topological dimension based on the cover of an object is first and foremost a subset of \(\mathbb{R}^n\). In this context, we prefer an equivalent definition of dimension. In Euclid's "elements", the concept of dimension is already defined, implicitly and inductively. A figure is said to be one-dimensional, if its boundary is made up of points; two-dimensional, if its border is composed of curves and three-dimensional, if its border is composed of surfaces [Mandelbrot, M., J. S. Geronimo, A., N. Harrington: 1984].

Topological dimension. Hermann Weyl illustrates the concept of dimension in the following terms: We say that space is three-dimensional because the walls of a prison are two-dimensional [Mandelbrot, M: 1978].

Pricing topology

The construction of the topological dimension can be based on the idea of generalizing the concept that the dimension of a ball is three while the dimension of the sphere that limits it is two: dimension of a set \(X\) from the dimension of its boundary \(\partial X\). On the other hand, a fractal object is first and foremost a subset of \(\mathbb{R}^n\). In this context, we prefer an equivalent definition of topological dimension based on the cover dimension, a concept that plays an important role in the definition of fractal dimension.

Fractal media [Frame, M., Philip, A., G., D, Robucci, A: 1994]:

\[
D(\alpha; x) \approx \frac{\alpha x}{12} \left(1 + \frac{\alpha x}{\sqrt{24}}\right)^{-3}
\]

Multifractal evidence:

\[
\beta_\alpha(r) = \left[ \frac{K \exp(-2(r/\sigma - 1))}{r/\sigma} \right] \quad \frac{r/\sigma > 1}{r/\sigma < 1}
\]
Positive layer:

\[ K = \frac{\beta Q}{\epsilon \sigma(1+\frac{z^2}{2})} \quad (4.1) \]

Negative layer:

\[ z^2 = -\frac{4\pi \sigma^2}{\epsilon k B T} \sum_{l=1}^{n} A_l^2 \quad (4.2) \]

Coating dimension. Let us consider a subset \( S \) of \( \mathbb{R}^n \). An open cover of \( S \) is any collection of open sets whose meeting contains the set \( S \). An open refinement of a ‘of the cover open \( a \) is another cover such that each open \( A \in a \) is included in some open \( A \in a \). In some sense, an open refinement to ‘of \( S \) provides a “more detailed” coating of \( S \) than \( a \).

Repellent moment- COVID-19 / Ex Ante:

\[ c[r, n] = \ln n + f \left[ (r - r') \left[ n(r') - n_b \right] \right] dr' \quad (4.2.1) \]

Recursively negative moment- COVID-19 / Apriori:

\[ G_n(r) = \exp \left[ -\beta \psi(r) + \int c_b \left[ (r - r') \right] n(r') - n_b \right] dr' \quad (4.2.2) \]

We say that \( a \) is an open cover of order \( k \) of the set \( S \), if, whatever \( x \in S \), \( x \) belongs to a maximum of \( k \) open of the cover \( a \). The set \( S \) has cover dimension (topological dimension) \( n \), if any open cover \( a \) of \( S \) admits an open refinement of order \( n + 1 \), but not of order \( n \).

Moment of Hope- COVID-19 / Ex Post:

\[ h(r) = c_b(r) + n_b \int c_b \left[ (r - r') \right] h(r') dr' \quad (4.2.3) \]

In the case of a segment divided into three equal parts; \( d = 1, M = 3 \rightarrow N = 3 \), a surface divided into three parts each side [Frame, M., Martino, W; 2010]; \( d = 2, M = 3 \rightarrow N = 9 \) and a cube3, dividing each side into three parts; \( d = 3, M = 3 \rightarrow N = 27 \), the capacity dimension allows evaluating the dimension of geometrically irregular objects \( \frac{l}{2n} \). Let’s consider the price \( P \), with respect to max (Z) profit margin for the investor.

\[ P' n(0) = I = \frac{l}{2n} \int_{c_r}^{\infty} \frac{p_n(z) dz}{z^2} \quad (5) \]

Assuming that \( d = \log (pN) / \log (pN) \), the smaller the radius, the greater the number of necessary circles or parts (n), from where \( n = 1 / r \). Hence \( d = \log (N) / \log (1 / z) \). Instead of counting the resulting self-similar parts (N), the number of circles \( N(z) \) will be counted; where the capacity dimension is the value of \( \log N(z) / \log (1 / z) \) when \( r \) tends to 0.

\[ a_2 = \frac{l}{2n} \int_{c_r}^{\infty} \frac{p_n(z) dz}{z^3} \quad (6) \]

Price skewed by COVID-19 risk

There are different ways to determine the price function:

Cover Dimension: The smallest number of sets needed to cover the object is calculated, which can overlap. If each point of the object is covered by no more than \( G \) sets then the coverage dimension is \( d = G-1 \).

\[ G_n(z) = z + \alpha_2 z^2 + \cdots + \alpha_n z^n \quad (4.2.1) \]

\[ G_n(z) = z + \beta_2 z^2 + \cdots + \beta_n z^n \]

\[ G_n = \lambda \pi_n + \mu \theta_n = z + b_2 z^2 + \cdots + \beta_n z^n \quad (7) \]

Iterative Dimension: It is based on the fact that the edges of the space of dimension \( D \) have dimension \( d-1 \), thus, every three-dimensional volume may be surrounded by two-dimensional planes [Maldelbrot, M., V. Jory, J., Herod, G. Passty: 1981]. To calculate it, the edges of the edges are searched until dimension 0 (point) is reached. The number of times the operation (H) is performed equals the dimension; \( d = H \).

\[ Max |H_n| \leq \lambda Max |H_n| + \mu Max |H_n| = D \]

\[ |H_n| \leq \lambda |H_n| + \mu |H_n| \quad (7.1) \]

Underlying Dimension (embedding): describes the space that contains the fractal object.

A function \( f: \mathbb{R} \rightarrow \mathbb{R} \) exhibits deterministic chaos [Maldelbrot, M; 1988] if it fulfills three properties according to:

\[ R_\alpha - \alpha_2 + (R_\beta - \alpha_3) + \cdots + (R_N - \alpha_N) z^{N-2} = 0 \quad (7.1.1) \]
Sensitivity to initial conditions: Arbitrarily close to each point \( x \), there is a point \( y \) with \( f^n (x) \) and \( f^m (y) \) iterating away.

Periodic points: Arbitrarily close to each point \( x \), there is a point \( y \) with \( f^m (y) = y \) for some \( m \).

Mix: For each pair of intervals \( I \) and \( J \), for some \( k \in \mathbb{N} (J) \) and overlap.

\[
i, j_n (z) = p_n (z) - \theta S_n (z)
\]

For there to be an aperiodic price signal, the possibility of maximization must be represented by a Harmonic or Fourier Series [Mandelbrot, M., J. Elton, D. Hardin: 1989], it must respect the Dirichlet conditions 1:

1) That it has a finite number of discontinuities in period \( T \), if it is discontinuous with the rotation vertices of the circulation of shares:

\[
M_{\text{v}, 1} \mu_{1} \mu_{3} \mu_{4} = \sum \nu M_{\nu, 1} \mu_{1} \mu_{3} \mu_{4} + M_{\nu, 2} \mu_{1} \mu_{3} \mu_{4} = 0
\] (8.1)

\[
M_{\text{v}, 2} \mu_{1} \mu_{3} \mu_{4} = \frac{\phi_{\nu} p_{1}^{j} g_{1}^{3} \mu_{1} \mu_{3} \mu_{4}}{2 \omega_{1} g_{2}^{3} \mu_{1} \mu_{3} \mu_{4}} \sum_{d=1}^{\delta} \nu M_{\nu, d, l} \mu_{1} \mu_{3} \mu_{4}
\] (8.2)

ii) The mean value in period \( T \), be finite in your price escape bubbles:

\[
T_{\nu, 1} \mu_{1} \mu_{3} \mu_{4} = \Gamma_{\nu, 1} \mu_{1} \mu_{3} \mu_{4} (p_1 - p_2 - p_3 - k - p_1 - p_2 - p_3, k - p_1 - p_2 - p_3)
\] (8.2.1)

\[
x_{p} g_{1}^{3} \mu_{1} \mu_{3} \mu_{4} \Gamma_{\nu, 1} \mu_{1} \mu_{3} \mu_{4} (p_2, k - p_1 - p_2 - p_3, k - p_1 - p_2 - p_3)
\] (8.2.2)

\[
x_{p} g_{1}^{3} \mu_{1} \mu_{3} \mu_{4} \Gamma_{\nu, 1} \mu_{1} \mu_{3} \mu_{4} (p_2, k - p_1 - p_2 - p_3, k - p_1 - p_2 - p_3)
\] (8.2.3)

Due to orthogonality, each term on the right hand side of the last equation is zero, except when \( m \neq n \).

The price coefficients that we are looking for are the ranges (Ex Post price - Ex Ante price) and divide them by the number of companies in the Mexican capital market.2

\[
E_{i} (z) = \frac{z}{p_{v}} + \cdots + \frac{z}{p_{n}}
\] (9)

\[
\left| \frac{f_{v} - z}{z^{2}} \right| - R_{p-2} < \varepsilon_{v}
\]

Through this tool that allows the approach to the study of the fractal constraint to the realization of the stock market practice regarding its prices with fractal noise:

\[
\Delta_{v, 1} = (k - m_{v}^{2}) \Gamma_{k - p_1 - p_2 - p_3, k - p_1 - p_2 - p_3} (k - p_1 - p_2 - p_3, k - p_1 - p_2 - p_3)
\] (9.1)

\[
\Delta_{v, 1,1} = (k - m_{v}^{2}) \Gamma_{k - p_1 - p_2 - p_3, k - p_1 - p_2 - p_3} (k - p_1 - p_2 - p_3, k - p_1 - p_2 - p_3)
\] (9.2)

\[
\Delta_{v, 2,1} = (k - m_{v}^{2}) \Gamma_{k - p_1 - p_2 - p_3, k - p_1 - p_2 - p_3} (k - p_1 - p_2 - p_3, k - p_1 - p_2 - p_3)
\] (9.3)

\[
M_{0} \mu_{1} \mu_{3} \mu_{4} + M_{\nu_{1}} \mu_{1} \mu_{3} \mu_{4} + M_{\nu_{2}} \mu_{1} \mu_{3} \mu_{4}
\] (10)

1 By multiplying the above equation by \( \phi_{\mu} (x) \), integrating in the interval \([a, b]\) of the Ex Post and Ex Ante Prices, we obtain:

\[
\int_{-p}^{p} f(x) \phi_{\mu} (x) dx = c_{0} + c_{1} \int_{-p}^{p} \phi_{\mu} (x) dx + \cdots + c_{n} \int_{-p}^{p} \phi_{\mu} (x) \phi_{\mu} (x) dx + \cdots
\]

2 We find the finite price and smooth it with the (sin) of (x-h) in F & V:

\[
\begin{align*}
\tilde{x} + \tilde{x} &= ax + bx^{3} + k(x - h) \oplus (x - h) = F \sin(wt) \\
F(x) &= \frac{\sin(\alpha x + bx^{3} + k(x - h) \oplus (x - h))}{2} \\
V(x) &= \frac{ax^{2} + bx^{3} + k(x - h) \oplus (x - h)}{2} \\
F \rightarrow & F \phi \in \mathbb{E}^{2} \quad \phi \rightarrow \mathbb{E}^{2}
\end{align*}
\]

Evidence of noise:

\[
T^{\mu,\nu}_{\chi_1,\chi_2} = \int \frac{dT\chi_1 T\chi_2}{(2\pi)^{-1} \Delta T\chi_1,\chi_2} \tag{11}
\]

\[
T^{\mu,\nu}_{\chi_1,\chi_2} = \frac{\Gamma_{\chi_1,\chi_2}}{\Gamma_{\chi_1,\chi_2}}(\frac{k + p_1 + p_2}{k - p_3 - p_1}) \tag{11.1}
\]

\[
T^{\mu,\nu}_{\chi_1,\chi_2} = \frac{\Gamma_{\chi_1,\chi_2}}{\Gamma_{\chi_1,\chi_2}}(\frac{k + p_1 + p_2}{k + p_1 + p_2} - k - p_1 - p_2 - p_3) \tag{11.2}
\]

\[
\mathcal{M}^{\mu,\nu}_{\chi_1,\chi_2} = \frac{1}{\pi^2} \sum_{i=1}^{\infty} F^i_\chi_1 F^i_\chi_2 \tag{12}
\]

\[
F^i_\chi_1 = F^{i+1}_\chi_1 + F^{i+1}_\chi_2 + F^{i+1}_\chi_2 \tag{13}
\]

We proceed in an analogous way to the case \( p = 1 \), but conveniently changing the Trading Volume of each share and thus continue the maximization of utility.

\[
\|f_v - P_n\| < \epsilon_v
\]

\[
E_t = z + z^2 R_{\epsilon_2 - \epsilon_1}
\]

\[
\max|P_n| < \rho_n \epsilon_v
\]

\[
E_t = \max|R_p| \leq \max|P_n| + \epsilon_v
\]

In each transaction there is a probability that the price will change and therefore leave the holding, and after a certain time horizon, there is a total change in the price. We obtained the price change (since the cumulative distribution obeys an inverse cubic law, the probability distribution function, by differentiation) and obeys an inverse (fourth-moment) quartic law [Frame, M., Neger, N: 2010].

\[
\mu < \rho_v + \epsilon_v \tag{14}
\]

\[
\mu \leq \rho
\]

\[
\rho = \lim_{n \to 0} E_t
\]

\[
\max|\varphi| = \lim_{p \to 0} \max|E_{r_\varphi}|
\]

This means that there is no characteristic scale for the diffusion of prices, because if it is diffusing around a medium with the bounding limit [Mandelbrot, B. B: 1983] that by itself is changing (like the economic universe in which we live), then the laws of diffusion change and, in particular, adopt a form of free scale.

Returns in the three times:

\[
h_i(r_{12}) = c_i(r_{12}) + \sum_{n=1}^{\infty} \rho_n \int d^3r \sum_{i=1}^{\infty} f_i\chi_1,\chi_2 \tag{14.1.1}
\]

\[
c_i(r) = -\beta u_i(r) + h_i(r) - \ln \left(1 + h_i(r)\right) + b_i(r) \tag{14.1.2}
\]

\[
R = \begin{pmatrix} R_A & 0 \\ 0 & R_B \end{pmatrix} \tag{15}
\]

Separation of fractal formation matrices:

\[
X(k) = \begin{pmatrix} X_{AA}(k) & X_{AB}(k) \\ X_{BA}(k) & X_{BB}(k) \end{pmatrix} \tag{16}
\]

North Bound:

\[
H_{AA}(k) = c_{AA}(k) R_A H_{AA}(k) + c_{AB}(k) R_B H_{AB}(k)
\]

South Bound:

\[
H_{AB}(k) = c_{AB}(k) R_A H_{AB}(k) + c_{AB}(k) R_B H_{AB}(k)
\]

East Bound:

\[
H_{BA}(k) = c_{BA}(k) R_A H_{BA}(k) + c_{BB}(k) R_B H_{BA}(k)
\]

West Bound:

\[
H_{BB}(k) = c_{BB}(k) R_A H_{BB}(k) + c_{BB}(k) R_B H_{BB}(k)
\]

Price shadow effect of COVID-19

Furthermore, the exponents of the index probabilities and volatility appear to be analogous to the exponents in a critical phenomenon, in the sense that they appear to be related in interesting ways of maximizing stock market space for the holding life of the stock within the market.

\[
\max|E_{\epsilon_2} | < E_{\epsilon_2} \tag{17}
\]

\[
\max|E_{\epsilon_2} | < E_{\epsilon_2} \tag{18}
\]
The algorithm of a Fractal [Mandelbrot, M., G. Turchetti: 1981] is the plot of the values of the orbit in order. That is, it is the graph of the points that maximizes all the possibilities of iteration of the prices \((0, x_0), (1, x_1), (2, x_2), \ldots\). When many points are represented by the market prices, the order can be valued by drawing lines that connect successive points by their range.

$$R_p(z) \leq I + \epsilon_n \quad (19)$$

$$x_t = T(t\psi(x_t) + (1 - t)x_t, \epsilon (0,1))$$

This is one of the most common ways to visualize the temporal patterns [Frame, M., Mandelbrot, BB: 2009] of stock prices and it is obtained first by dividing the range (maximum price / minimum price) represented vertically to be compatible with the iteration of the holding price.\(^3\)

Each point of the orbit belongs to some box (or period of time-\(K_0\)), and as follows in the orbit (market trend-\(K_{(n + 1)}\)) [Mandelbrot, M., G. Turchetti: 1982], increases each point on the horizontal line and gives us an approximate measure of the amount of time in holding that spends in the orbit of each price region.

Minifractalize: Delete with logarithms the price ranges to a fractal form, decreasing its volume of operation in the market and size but keeping the purchase or sale of shares identical \((\psi - Iq) \leq 0, \text{Min } \forall \epsilon FMin(T)\)

Dephractalize: Minimize or maximize until the externalities of financial risk are lost at a fractal price due to lack or collapse of the elements of bookkeeping within the market \(K_0 = \{q\}\)

Maxifractalize: Add buy and sell movements to an initial fractal form until the price and operation volume increase, keeping the stock market operation the same \(K_{n+1} = (K_n \cup T(K_n) \cup \psi(K_n))\)

Conclusions

All the tools used in this article have a single purpose: to detect and measure price trends to establish and manage buying and selling operations within a certain BMV market, so we use geographic systems (with their respective degrees), together with the shares that are in the holding.

All real complex systems generally exhibit scale invariance, that is, their behavior does not change due to rescaling [Mandelbrot, M: 1975] of the variables that govern their market price dynamics \(K = U_nK_n\)

$$||x_t - p|| = T(t\psi(x_t) + (1 - t)x_t) - T_p||$$

The stochastic price path becomes unstable, this is where multifractals outperform the informal Euclidean representation [Frame, M., Neger, N: 2008]. While with the Euclidean premise it is not possible to answer many questions about price prospecting phenomena and its positive or negative recursion is possible to represent an infinite number of irregular, non-linear shapes [Mandelbrot, M: 1981], being suitable to represent prices about the shares \(\leq t(\psi(x_t) - \psi(p)) + (1 - t)(x_t - p) + t(\psi(p) - p)||

$$||x_t - p|| \leq \frac{t}{1-p}||\psi(p) - p||$$

Limiting the limit in \(k (0)\) - COVID19 Ex Post:

$$\lim_{k \to \infty} \left(\psi(q) - q, J(x_{nk} - q)\right) = \Gamma$$

$$\Phi(||x_{nk} - \bar{x}||) \leq \Phi(||t_{nk}(\psi(x_{nk}) - \psi(\bar{x})) + (1 - t_{nk})(x_{nk} - \bar{x}) + t_{nk}(\psi(\bar{x}) - \bar{x})||)$$

$$\leq \Phi(||t_{nk}(\psi(x_{nk}) - \psi(\bar{x})) + (1 - t_{nk})(x_{nk} - \bar{x})$$

$$+ t_{nk}\delta_k + t_{nk}(\psi(\bar{x}) - \bar{x})J_{\phi}(x_{nk} - \bar{x})$$

$$\leq (1 - (1 - p)t_{nk}\Phi(||x_{nk} - \bar{x}||) + t_{nk}\delta_k + t_{nk}(\psi(\bar{x}) - \bar{x})J_{\phi}(x_{nk} - \bar{x})$$

Regarding the narrowing of its limits in the price range, we obtain:

$$\sum_{n=1}^{p} \left( a_n \int f_p \cos \frac{\pi}{p} x \cos \frac{\pi}{p} x dx + b_n \int f_p \cos \frac{\pi}{p} x \sin \frac{\pi}{p} x dx \right)$$

\(^3\) If we get all the ranges of stock prices.

$$\int_{-p}^{p} f_p (x) \cos \frac{\pi}{p} x dx = \frac{a_n}{2} \int_{-p}^{p} \cos \frac{\pi}{p} x dx$$

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\[
\lim_{k \to \infty} \Phi \left( \left| x_{n_k} - \bar{x} \right| \right) \leq \lim \sup_{k \to \infty} \left( \delta_k + (\psi(\bar{x}) - \phi(x_{n_k} - \bar{x}) \right)
\]

Representation of finite recursion to the Brownian equilibrium [Mandelbrot, B. B: 1967] of stock market operation:

\[
n \geq 0
\]

\[
\left| \psi(x_n) - x_n \right| \leq (1 + p) \left| x_n - x_0 \right|
\]

\[
\left| x_n - T x_n \right| \leq \left| x_{n+1} - x_n \right| + \alpha_n \left| \psi(x_n) - \bar{x} \right|
\]

The path of total recursion would be simulated in the Mexican capital market considering the temporality of the COVID-19 risk in Mexico, considered as an important element of current public policy.

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