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## Presentation of the Content

In the first chapter we present, *Predictive relationship of knowledge management and business innovation: A model based on PLS structural equations* by VILLALOBOS-ALONZO, María de los Ángeles & ROMO-GONZÁLEZ, Ana Eugenia with adscription in the Universidad Tecnológica de Jalisco e Instituto Superior de Investigación y Docencia para el Magisterio, as a second article we present, *Analysis of the Perception of Quality in Service in a Higher Education Institution with the use of the ModelServQual* by LIMON-VALENCIA, Luis Alberto, GRIJALVA-TAPIA, Juan, RUIZ-CASTRO, Manuela and HINOJOSA-TAOMORI, Karina Alejandra with adscription in the Instituto Tecnológico Superior de Cajeme as the following article we present *Humidity and comparative analysis of durability index in pellet of balanced foods for birds* by VALLEJO-SARTORIUS, Irma, RENDON-SANDOVAL, Leticia and GUTIERREZ-PEÑA, Esteban with affiliation at the Instituto Tecnológico Superior de Huatusco as next article we present *Analytical proofs of expected value and variance of the main functions of continuous probability distributions, with examples of their applications* by VILLAGÓMEZ-MÉNDEZ, Juan, HERRERA-MIRANDA, Miguel Apolonio, HERRERA-MIRANDA, Israel and CASTILLO-MEDINA, Jorge Antonio with affiliation at the Universidad Autónoma de Guerrero.

## Content

# Predictive relationship of knowledge management and business innovation: A model based on PLS structural equations

# Relación predictiva de la gestión del conocimiento y la innovación empresarial: Un modelo basado en ecuaciones estructurales PLS

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**Abstract**

In this study we describe flexible modeling using structural equations of partial least squares (PLS) based on the analysis of variance, to predict the dependency relationships of a theoretical model supported by internal and external conditions for the development of innovation capabilities that they include the endogenous variables; organizational culture, exploitation innovation, exploration and ambidestreza, innovative performance, competitiveness, University-Business collaboration and business innovation, and its relationship with the exogenous variable; knowledge management practices. In the method, three phases were applied; the consideration of the theory and previous investigations for the construction of the conceptual model, the application of the measurement model related to the attributes of validity and reliability of the constructs, the structural model that evaluates the weight and the magnitude of the relationships between variables. Out of the main results, values higher than the minimum variance value explained were obtained, which is recommended to be Falk $\geq$ 0.10 and a model adjustment value greater than 0.50, thereby inferring the existence of a positive relationship of predictive interdependence and significant variance of the variables, which favor the creation of product, service or process innovations in companies.

**Structural equation modeling, Knowledge management, Business innovation**

**Resumen**

En este estudio se describe la modelización flexible mediante ecuaciones estructurales de mínimos cuadrados parciales (PLS) basado en el análisis de la varianza, para predecir las relaciones de dependencia de un modelo teórico sustentado en condiciones internas y externa para el desarrollo de capacidades de innovación que comprenden las variables endógenas; cultura organizacional, innovación de explotación, exploración y ambidestreza, desempeño innovador, competitividad, colaboración Universidad-Empresa e innovación empresarial, y su relación con la variable exógena; prácticas de la gestión del conocimiento. En el método, se aplicaron tres fases; la consideración de la teoría e investigaciones previas para la construcción del modelo conceptual, la aplicación del modelo de medida relacionado con los atributos de validez y fiabilidad de los constructos, el modelo estructural que evalúa el peso y la magnitud de las relaciones entre variables. De los resultados principales, se obtuvieron valores superiores al valor de varianza mínimo explicado, que se recomienda que sea Falk $\geq$ 0,10 y un valor de ajuste del modelo mayor a 0,50, lo que infiere la existencia de una relación positiva de interdependencia predictiva y una varianza significativa de variables que favorecen la creación de innovaciones de productos, servicios o procesos en las empresas.

**Modelación de ecuaciones estructurales, Gestión del conocimiento, Innovación empresarial**

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## Introduction

In recent years, the Models of Structural Equations (MEE) have become a “multivalent analysis tool widely used in the field of economics and social sciences” (Céspedes and Sánchez, 1996).

These models Fornell (1982) calls them as "second generation, since they help link data and theory", allowing to expose a scientific explanation that goes beyond association or description.

The MEEs are intended to make multiple regressions between latent variables and variables. However, not all models of structural equations are based on covariance and factor analysis.

The partial least squares (PLS) analysis technique “aims at the prediction of latent variables and is based on the estimation of ordinary least squares (OLS) and on the analysis of main components, path and regression analysis” (Cepeda y Roldan, 2007). In addition, modeling relationships between multiple predictor variables (independent, exogenous) and criteria variables (dependent or endogenous).

The PLS technique was developed by the Swedish professor Herman Wold, at first it was called NIPALS (Nonlinear Iterative Partial Least Squares in 1973). It is currently known as PLS and its basic design was completed in 1997.

The PLS technique is suitable for predictive purposes (Chin, Marcolin, and Newsted, 2003), oriented “to predictive causal analysis in highly complex situations, but with a poorly developed knowledge or applied in a new context” (exploratory nature) ( Wold, 1979; Wong, 2006), so it is "used to develop a nascent theory" (Barclay, Higgins, and Thompson, 1995). Its use has been extended in the areas of knowledge oriented to the organization of companies.

In this context, PLS represents a system of mathematical and statistical data analysis that adapts to the present conditions of the economic-administrative and social sciences "proposing a flexible modeling" (Wold, 1980) "creating optimal linear predictive relationships between variables" (Cepeda and Roldan, 2007).

In this sense of least squares, this means that, "given the data and the model, independent variables become the best possible predictor variables, and dependent variables become the best criteria or predicted variables" (Falk and Miller, 1992).

Flexible modeling can be used, even if one or more of the following conditions and circumstances concur according to Falk and Miller, (1992) in Cepeda and Roldan, (2007):

| **Conditions that do not limit the use of PLS** | |
|---|---|
| **Conditions** | **Elements** |
| Theoretical | The hypothesis is derived from a macro level theory in which not all variables are known. The relationships between theoretical constructs and their manifestations are vague.<br>The relationship between constructs is conjectural. |
| Measure | Some or all of the manifest variables are categories or present different levels of measurement.<br>The manifest variables have some degree of unreliability.<br>The residuals of latent and manifest variables are correlated (Heterocedasticity). |
| From Distribution | The data comes from unknown or non-normal distributions. |
| Practices | It is used in non-experimental research. A large number of latent and manifest variables are modeled. They are available, either too many cases, or a small number. |

**Table 1** PLS conditions. Source: Falk and Miller, (1992).

For the application of the PLS technique, the exploratory and relational nature of variables should be considered. According to Chin, (1998) the analysis of structural equations responds to two phases: the measurement model and the structural model.

The "measurement model" analyzes whether the theoretical concepts are measured correctly through the observed variables.

This analysis is performed regarding the attributes of validity (it measures what you really want to measure) and reliability (it does so in a stable and consistent way).

On the other hand, the “structural model” evaluates the weight and magnitude of the relationships between variables.

The terms commonly used according to Barroso, (2007) have to do with the “graphic representation of the data, the theoretical constructs, latent or unobservable variable (circle) and the indicators, measures, manifest or observable variables (table)”.

The measures for the analysis of the predictive relationship of variables were obtained through the process suggested by Anderson, Rungtusanatham, Schroeder, & Devaraj (1995): descriptive analysis, exploratory analysis (factor analysis of main components and reliability). The measurement model included theoretical modeling through hypotheses, statistical tests and the design of the model according to the recommendations of several authors (Cepeda and Roldan, (2007); Temme (2006), Chin, (1998) and Fornell, ( 1982), considering that: The research hypotheses consider a positive relationship between each construct of the variables according to the bivariate analysis.

a) Indicators that were validated under content, criteria and construct are used.
b) There is no evidence of collinearity, since they have reflective-formative indicators, which must be highly correlated and with high levels of internal consistency using Cronbach's Alpha and composite reliability (Chin, 1998).

## Method

Due to the nature of the hypothesis (relational) and the exploratory nature of the investigation, the use of PLS is used to perform analyzes between the constructs, which means that the analysis performed is neither causal nor confirmatory.

HE: There is a positive relationship of predictive and significant interdependence of the variances of the endogenous variables that make up the internal conditions (innovative performance, innovation of exploration, exploitation and ambidestreza and organizational culture) and external conditions (Company-University collaboration, competitiveness and indicators of business innovation) and the exogenous variable knowledge management practices in the structural model for the development of innovation capabilities (Figure 1).

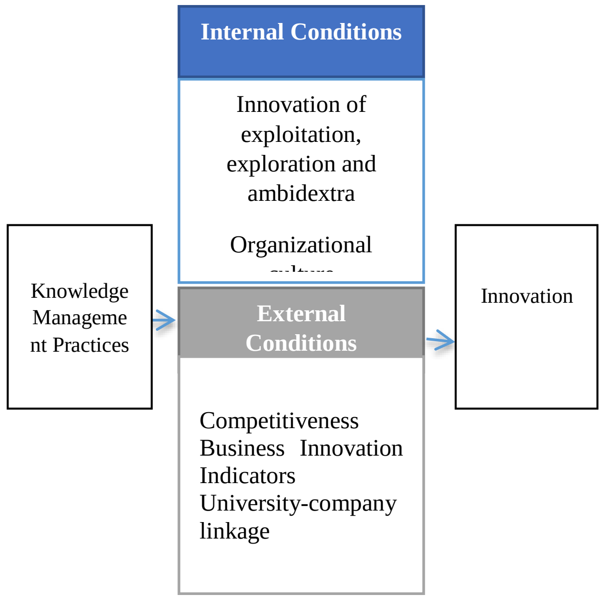


**Figure 1** Relation of the general multivariate hypothesis

The study is of a transversal nature, so the results should be treated with caution since “due to the time factor, it is difficult to establish causality, so the data will only be used to investigate relationships between constructs (Im, Pesaran & Shin 2003).

For the PLS analysis, 7 constructs are included: EEA Innovation (EEAI) with 2 indicators, Organizational Culture (CO) with 3 indicators, Innovative Performance (DI), Competitiveness (Cm), IE Indicators (InI) and EU collaboration (VE-U) and knowledge management practices with 4 indicators each, giving a total of 24. For the PLS analysis the SmartPLS 3.0 software was used.

## Results

To determine the predictive relationship of the study, the process of the two phases of the analysis of structural equations was followed: A) the measurement model and B) structural.

### A. Measure Model

The analysis of the measurement model according to Cepeda and Roldan, (2007), Barroso, (2007), Chin, (1998) and Fornell, (1982) implies the analysis of individual reliability of the item, internal consistency or reliability of a scale, convergent validity analysis and discriminate validity.

| Reliability: Cronbach's Alpha | | | |
|---|---|---|---|
| **Variables** | **Component** | **Question Code** | ***α* item** |
| Knowledge management practices pc = 0.876 α = 0.779 | Available storage systems | PG! | 0,735 |
| | Shared knowledge spaces | PG2 | 0,669 |
| | Collaborative Learning | PG3 | 0,673 |
| | Knowledge application | PG4 | 0,902 |
| | | DI5 | 0,932 |
| Terms internal pc = 0.953 α = 0.811 | Innovative Performance pc = 0.873 α = 0.831 | DI6 | 0,699 |
| | | DI7 | 0,808 |
| | | DI8 | 0,674 |
| | Exploitation, exploration and ambidestreza innovation pc = 0.638 α = 0.626 | IEEA9 | 0,620 |
| | | IEEA10 | 0,639 |
| | Organization (Culture)$p_c$=0,953 α=0,944 | OC11 | 0,866 |
| | | OC12 | 0,896 |
| | | 0C13 | 0,970 |
| External Conditions pc = 0.940 α = 0.922 | Company-University Collaboration pc = 0.737 α = 0.629 | CE-U14 | 0,646 |
| | | CE-U15 | 0,794 |
| | | CE-U16 | 0,622 |
| | | CE-U17 | 0,629 |
| | Competitiveness pc = 0.890 α = 0.878 | Cm18 | 0,831 |
| | | Cm19 | 0,843 |
| | | Cm20 | 0,873 |
| | | Cm21 | 0,825 |
| | Business Innovation Indicators pc = 0.979 α = 0.978 | InIE22 | 0,961 |
| | | InIE23 | 0,983 |
| | | InIE23 | 0,967 |
| | | InIE25 | 0,972 |

**Table 2** Analysis of the PLS average model

Reliability with Cronbach's Alpha presupposes a priori that each indicator of a construct contributes in the same way, the loads are set in the unit (Barclay, Higgins, and Thompson, 1995). The criteria used for the interpretation of the reliability coefficient are: less than 0.6 (low); 0.61 to 0.70 (appropriate); 0.71 to 0.80 (good); greater than 0.80 (high) (Nunnally, 1978). It presents the reliability of the constructs (Table 2).

Considering the results of table 2, in general the constructs are probos, since they have a composite reliability greater than 0.6.1.

Now convergent validity is carried out, which describes "if the different items intended to measure a concept or construct really measure the same, then the items will be significant and highly correlated" (Cepeda and Roldan, 2007). This is obtained through the mean extracted variance (AEV) shown in Tables 3.1 and 3.2.

| **Convergent validity** | | |
|---|---|---|
| Component | | AEV |
| Latent variable Knowledge Management Practice | Knowledge Management Practice | |
| | ,527 | ,586 |

**Table 3.1** Convergent validity matrix

| **Convergent Validity** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Component | | Internal conditions | | | External conditions | | | AEV |
| | | **1** | **2** | **3** | **4** | **5** | **6** | |
| **Terms internal** | 1.DI | **0,867** | | | | | | 0,752 |
| | 2. EEAI | -0,020 | **0,904** | | | | | 0,817 |
| | 3. Or | 0,711 | -0,261 | **0,955** | | | | 0,913 |
| **Externa Conditio** | 4.CE-U | | | | **0,685** | | | 0,569 |
| | 5. Com | | | | 0,835 | **0,859** | | 0,918 |
| | 6. IIE | | | | 0,739 | 0,783 | **0,970** | 0,941 |

**Table 3.2** Convergent validity matrix

In general, the constructs that make up the model, obtained the value of Analysis of Extracted Variance (AEV) greater than 0.50 (Fornell, 1982) complying with the convergent validity condition.

The second refers to discriminant validity (Table 4.1), which is the variance shared between the construct and its measures. This measure should be greater than the variance shared between the construct with the other constructs (square correlation between the two constructs). It will be necessary to demonstrate that each indicator relates better to its own construct than to another.

It is observed in Table 4.1 that the loads of the constructs that make up the internal conditions for the development of innovation capacities are better related to their own construct.

| **Discriminant validity** | | | | |
|---|---|---|---|---|
| **Components** | **Innovación EEAI** | **Desempeño Innovador** | **Organización (Cultura hacia la innovación)** | **Prácticas de gestión del conocimiento** |
| **DI5** | -0.162 | **0.565** | 0.942 | 0.281 |
| **DI6** | 0.040 | **0.973** | 0.589 | 0.391 |
| **DI7** | 0.157 | **0.874** | 0.295 | 0.286 |
| **DI8** | -0.087 | **0.990** | 0.665 | 0.455 |
| **IEEA10** | **0.887** | -0.126 | -0.260 | -0.423 |
| **IEEA9** | **0.920** | 0.074 | -0.217 | -0.498 |
| **OC11** | -0.375 | 0.569 | **0.978** | 0.434 |
| **OC12** | -0.162 | 0.565 | **0.942** | 0.281 |
| **OC13** | -0.186 | 0.856 | **0.946** | 0.462 |
| **PG1** | -0.524 | 0.395 | 0.421 | **0.989** |
| **PG2** | -0.274 | 0.424 | 0.327 | **0.737** |
| **PG3** | -0.335 | 0.402 | 0.334 | **0.778** |
| **PG4** | -0.133 | 0.263 | 0.310 | **0.500** |

**Table 4.1** Matrix of crossed loads and divergent validity: Internal

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### Conditions

*** Calculations made in SmartPLS3.0**.

In table 4.2 it is inferred that the loads of the constructs that make up the external conditions for the development of innovation capacities are better related to their own construct.

For the interpretation of the divergent validity of the level of the charges, "the ranges between 0.50 and 0.60 as acceptable" will be taken (Chin, 1998). It is observed in tables 3.1, 3.2, 4.1 and 4.2 that the loads obtained are greater than 0.50. The results obtained support convergent and divergent validity.

The validity and reliability conditions are met in the measurement model, the second phase corresponding to the structural model will proceed.

| Discriminant validity | | | | |
|---|---|---|---|---|
| **Components** | **Company Collaboration** | **College Competitiveness** | **Innovation Indicators Business** | **Management Practices knowledge** |
| CE-U14 | 0.729 | 0.481 | 0.355 | 0.561 |
| CE-U15 | 0.516 | 0.275 | 0.290 | 0.263 |
| CE-U16 | 0.791 | 0.790 | 0.509 | 0.435 |
| CE-U17 | 0.671 | 0.698 | 0.885 | 0.412 |
| Cm18 | 0.741 | 0.791 | 0.698 | 0.325 |
| Cm19 | 0.781 | 0.922 | 0.927 | 0.614 |
| Cm20 | 0.685 | 0.889 | 0.571 | 0.682 |
| Cm21 | 0.707 | 0.830 | 0.478 | 0.390 |
| InIE22 | 0.745 | 0.780 | 0.991 | 0.477 |
| InIE23 | 0.678 | 0.714 | 0.945 | 0.446 |
| InIE24 | 0.741 | 0.786 | 0.978 | 0.488 |
| InIE25 | 0.702 | 0.754 | 0.966 | 0.474 |
| PG1 | 0.633 | 0.609 | 0.434 | 0.962 |
| PG2 | 0.477 | 0.503 | 0.463 | 0.815 |
| PG3 | 0.482 | 0.508 | 0.434 | 0.808 |
| PG4 | 0.342 | 0.356 | 0.274 | 0.555 |

**Table 4.2** Matrix of crossed loads and divergent validity: External Conditions

### Structural model

The objective of the structural model is to confirm "to what extent the relationships specified in the theoretical model are consistent with the data" (Real, Leal and Roldán, 2006). For the evaluation of the model, two basic $R^2$ indices and standardized path coefficients β will be used.

$R^2$ It indicates the amount of "variance of the construct that is explained by the model" (Cepeda and Roldán, 2007).

The endogenous variable ($R^2$) should be greater than or equal to 0.1 (Falk and Miller, 1992). "If the values of $R^2$ are less than 0.01, even if they are statistically insignificant, changes in the $R^2$ indicator can be explored to determine if the influence of a particular latent variable on a dependent construct has a substantive impact" (Chin, 1998).

The path coefficient or standardized regression weights is identified in the monogram by means of the arrows that link the constructs in the internal model (Cepeda and Roldán, 2007). To be considered significant results or path coefficients, they should reach a value of 0.2, and ideally be above 0.3 (Chin, 1998).

It is important to note that the magnitude of the indirect effects on a variable is estimated by multiplying the existing path coefficients along the line of the causal line between two related variables (Arbuckle, 2003).

The monograms were designed using Smart PLS of the Internal Conditions (Figure 2) and External with relation (Figure 3) to the latent variable of knowledge management practices, to identify the variances of the constructs and the strength of relationship.

DI5 Implement innovation projects that respect the environment. DI6 Products developed from the follow-up of customer needs. DI7 Opening of national markets. DI18 Increase in customers. IEEA9 Type of Innovation. IEEA10 Innovation management. OC11 Declared culture towards innovation. OC12 Leadership (senior management support). OC13 Applications and tools (software).

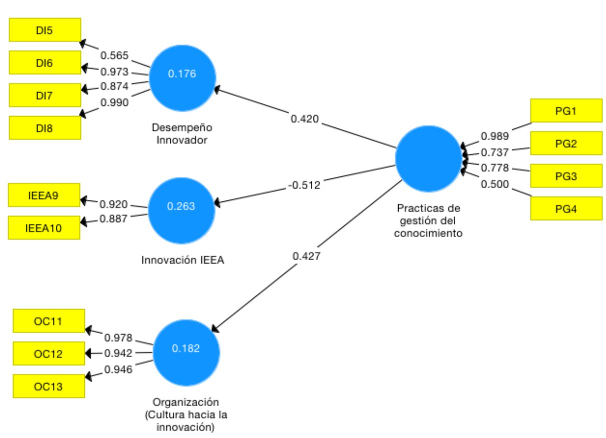


**Figure 2** Monogram of the Internal Conditions for the development of innovation capabilities (PLS)

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It can be seen in Figure 2, in general, the endogenous variables have a predictive relevance in the three constructs that integrate the internal conditions for the development of business innovation capabilities by obtaining a variance explained in Innovative Performance $R^2$ = 0.176, EEA Innovation $R^2$ = 0.263, Organizational (Culture) $R^2$ = 0.182, exceeding the minimum value of explained variance that is recommended (≥ 0.10 (Falk, 1992: 80).

CE-U14 Reason for collaboration with the University. CE-U15 Collaboration with the University and the government sector for the development of R + D + i projects. CE-U16 The response of the academy to the demands of consulting, research and innovation in the industrial sector. CE-U17 Knowledge transfer. Cm18 Productivity (annual increase according to goals). Cm19 Innovation strategy Expenditure on R + D + i / Sales). Cm20 Registered Patents. Cm21 Design of a financial plan for the development of innovation activities. InIE22 Training oriented towards the creation of innovation. InIE23 Participation of managers in innovation activities. InIE24 Collaboration networks of which the company is a part to identify opportunities for innovation. InIE25 The possibility of implementing an idea arising from the staff of your company, so that it becomes a product or service that is launched to the market.

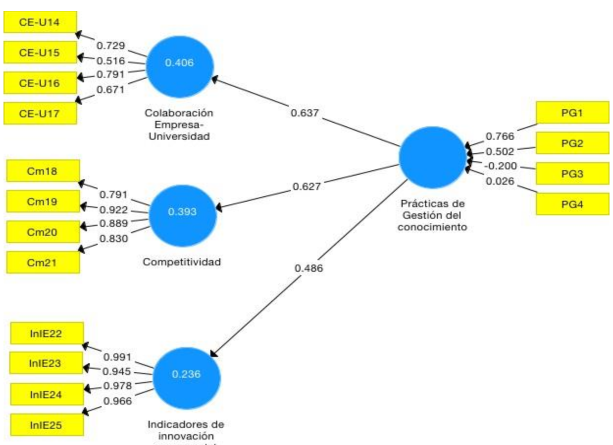


**Figure 3** Monogram of the External Conditions for the development of innovation capacities (PLS). Calculations made in SmartPLS3.0

In relation to the external conditions that integrate business innovation capabilities (Figure 3), they obtained the following results; Collaboration E-U $R^2$ = 0.406, Competitiveness $R^2$ = 0.393, and Business Innovation Indicators $R^2$ = 0.296, exceeding the minimum value of explained variance that is recommended (≥ 0.10 (Falk, 1992: 80).

The magnitude of the relationship effect between the variables that make up this study according to the path coefficient is shown in Table 5:

| Constructs | β | Level of Significance |
|---|---|---|
| PGC-Innovative Performance | 0, 420 | <0,3 |
| PGC-EEA Innovation | -0,512 | <0,3 |
| PGC-Organization (Culture towards Innovation) | 0, 427 | >0,3 |
| PGC-Competitiveness | 0, 637 | <0,3 |
| PGC-E-U Collaboration | 0, 627 | <0,3 |
| PGC-IE Indicators | 0,486 | <0,3 |

**Table 5** Path coefficient

With what can be verified that there is a positive relationship at an ideal level when exceeding 0.3 (Chin, 1998) between the prediction of the latent variable that directly and significantly influences the endogenous variables, except in the EEAI construct with influence in a way inverse.

Finally, mention that the PLS technique does not have an index that can calculate the goodness of fit of the model, so Tenenhaus, Vinzi, Chatelin and Lauro (2005: 173) propose a global evaluation criterion, which is calculated by the GoF formula. They are obtained by the root of GoF = AEV * $R^2$. For this investigation an adjustment index was obtained as follows:

GoF= √,802*,325=,510

Slightly higher than the recommended minimum (0.50). Once the different analyzes of the research model have been carried out, the results obtained from the PLS technique can be considered, the model reaches a predictive relevance in the six endogenous variables with $R^2$ higher than the minimum value of explained variance that is recommended (≥ to 0.10 (Falk, 1992: 80) and an adjustment value of the upper model 0.50 obtaining a value of 0.510 (Tenenhaus, 2005).

## Conclusions

The model based on structural equations denotes that there is a predictive and positive relationship in the deployment of processes that impact knowledge management and business innovation.

For the most part, there are positive and significant statistical relationships at a moderate level, which means that “the greater the relationship between the EEAI, CO and DI variables that make up the internal conditions and knowledge management practices in the organization, the greater the impact on the development of business innovation ”, since individual and organizational knowledge will be mobilized to create innovative processes, services or products that will mark a differentiating advantage for the organization from its competitors.

In general, there is a positive and significant statistical relationship at a high to moderate level, which means that “the greater the relationship between the variables Cm, InIE and VE-U that make up the external conditions with the knowledge management practices in the organization, greater impact on development in business innovation ”, since companies take advantage of the opportunities of the environment through collaboration or linking (competitors, customers, markets, society, government, university, business affiliations) to appropriate information and knowledge, and transform it in products, services or processes that allow them to survive and compete in dynamic and complex contexts.

Companies that wish to achieve greater organizational results must have valuable and inimitable resources and capabilities that can be a source of sustainable competitive advantages (Wernerfelt, 1984; Barney, 1986; Amit and Schoemaker, 1993).

There are several models of knowledge management that can be used by companies in the industrial, commercial and services sector for the generation and use of knowledge, among them, are the Taxonomy of models of knowledge management proposed by Barragan, (2009: 74).

The result of this research proposes a conceptual pattern marked in "scientific and technological models of knowledge management: whose purpose is the management of technological innovation and its purpose is to promote research and development within public or private organizations" ( Barragan, 2019: 75).

The knowledge management model to develop innovation capacities in the industrial sector (Villalobos, 2015: 221) is based on knowledge management practices that are developed in three dimensions; organization, R + D + i and the environment, which interact and feed back according to the predictive results shown in the monograms, which were obtained from the application of the ACIEAT survey to IT, Manufacturing, Automotive and Aerospace companies (Alonzo and González , 2015).

The efforts of the research academy are glimpsing new theoretical and practical approaches to the development and use of knowledge in companies, since it is evident that in an economy where the only certainty is uncertainty, the only source of lasting competitive advantage and Sure is knowledge (Nonaka, Takeuchi and Konno, 2000).

In this context, companies must generate their own source of knowledge and use it to produce innovations of process, product and service, flexible collaboration between the Academy-Government-Society-Industry, is and will be the key to the success of a nation with a vision for innovation.

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# Analysis of the Perception of Quality in Service in a Higher Education Institution with the use of the ModelServQual

# Análisis de la Percepción de la Calidad en el Servicio en una Institución de Educación Superior con el uso del Modelo ServQual

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**Abstract**

The main objective is to evaluate the quality of the service; likewise, to determine what are the essential aspects and measure the quality of service from the students. As expressed by Duque and Chaparro (2012), it is not recommended that the evaluation of the quality of the service only considers the point of view of executives and government agencies, since the perception of the most important client, the student, would be evaded. The method to follow is to determine the variables that will be studied and we establish the instrument to collect information, determine the population to which the study is directed, apply the instrument to the sample, present the results obtained, draw conclusions comparing it with the theoretical framework and with other studies; and finally, the recommendations of the findings. Among the main findings, it was possible to obtain the qualification of the service quality of the students in the period evaluated, the overall score reached is 5.18, interpreted as good service and representing 74%. It is relevant to consider that the characteristic with lower qualification, in this case, the appearance of physical facilities, equipment, personnel and communication material.

**Quality, Service, ServQual**

**Resumen**

El objetivo principal es evaluar la calidad del servicio; así mismo determinar cuáles son los aspectos más importantes y medir la calidad de servicio desde el punto de vista de los estudiantes. Como lo expresan Duque y Chaparro (2012), no es recomendable que la evaluación de la calidad del servicio solo considere el punto de vistas de directivos y organismos gubernamentales, ya que se estaría evadiendo la percepción del cliente más importante, el alumno. El método a seguir es determinar las variables que se estudiarán y se establece el instrumento para recolectar información, determinar la población a la cual se dirige el estudio, aplicar el instrumento a la muestra, presentar los resultados obtenidos, elaborar conclusiones comparándola con el marco teórico y con otros estudios; y por último se realizan las recomendaciones de los hallazgos. Entre los hallazgos principales se pudo obtener la calificación de la calidad del servicio de los estudiantes en el período evaluado, la puntuación general alcanzada es 5.18, interpretada como buen servicio y que representa un 74%. Es relevante tomar en cuenta que la característica con menor calificación, en esta caso, la apariencia de las instalaciones físicas, equipo, personal y material de comunicación.

**Calidad, Servicio, ServQual**

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## Introduction

Education is considered one of the most important tools for the development of humanity. According to the United Nations Educational, Scientific and Cultural Organization (UNESCO, 2017), education is a right of every person and that education must be of quality. And referring to educational quality, Edwards (1991), defines it as a judgment fixed to an educational product from the comparative point of view; it compares the observed reality with a desirable term. He adds that, to improve educational quality, it is necessary to use an approach from the point of view of the subject, that is, of the one who learns, the student.

The Mexican government, taking into account the need to achieve comprehensive development of the country, through the General Directorate of Higher Technological Education (DGEST, 2017), took on the task, in 2012, of updating the processes, plans and programs of study for the training of professionals, creating for this purpose, the Educational Model for the 21st Century: Training and Development of Professional Skills. In this document, it is specified that within its main guidelines, it is to increase the quality of educational services of the institutions of the National System of Technological Institutes.

Duque and Edison (2005) indicate that due to the characteristic of the intangibility of the services, their quality is measured, in many circumstances, subjectively. In their work, they define service quality as the difference between consumer expectations and their perception of the result. Also, Torres and Vásquez (2015), indicate that the common attributes of different methods of evaluating service quality are: tangible aspects, user care and service reliability. Service quality should be defined by the attributes that the customer expects from the service and by the amount he is willing to pay for that service. This is similar to the quality of physical products, where the attributes are easier to quantify.

As indicated by Reyes and Reyes (2012), the evaluation of the quality of the educational service at the higher level is carried out by different means: teacher evaluation, accreditation of degree programs, suggestion box, among others.

They add that the evaluation of the quality of the service in these institutions is of great importance, to have control of the processes and thus be able to improve their operation. They also describe that a tactic to increase the quality of the educational service is to satisfy the expectations of the students, making reference to the process that a company carries out to fulfil the expectations that its clients have.

In a study by Duque and Chaparro, they admit that an assessment of educational quality in a higher-level institution that does not consider the student's perception is to ignore the importance of the role played by the student in the process of continuous improvement (as quoted in Duque & Gómez, 2014, p. 183).

The coordination of the career under study states that according to interviews with their two predecessors in the position, which in sum cover ten years, no study has been conducted to find out what aspects students consider important when making a judgment on the quality of the service they receive from the institution.

This leads to the following questions addressed in this investigation: What are the dimensions that students in the Industrial Engineering career consider most important when evaluating the quality of the service they receive?

Considering the problem posed and the questions that this research seeks to answer, the general objective is to evaluate the quality of the service from the point of view of the students of an educational program. Looking for with this: First, to determine which are the most critical aspects, about the quality of service and to measure the quality of service that they perceive. The present study was carried out only on the students of an educational program. It will be limited to knowing the perceptions of those students about the service they offer using the SERVQUAL instrument. The study does not intend to develop, implement or test strategies to improve the quality of the educational service.

This article presents an analysis of the most important theoretical elements and foundations, the method of the work as well as in the analysis of the results obtained.

**Overview of services**

Services have become an essential product in the economy of most societies. According to the World Trade Organization (WTO), the rapid growth of the service sector makes it the generator of two-thirds of world output, which in turn generates one-third of employment. (World Trade Organization, 2017)

As explained by Lovelock, Reynoso, D'Andrea and Huete, (2004), it is crucial for managers to know the series of steps involved in producing a service, since clients regularly participate in generating it. Fernández and Bajac (2003) state that for a service to provide a valuable experience for the consumer, it must be correctly designed in all its components, among which they mention the operating processes. Just like the process of transforming raw materials into a finished product, the process into a service is equally or more relevant, since, in many situations, the same client participates in that process.

To visualize more broadly the types of processes of a service, Lovelock, Reynoso, D'Andrea and Huete, (2004), divide the processes in four categories: tangible acts directed to the body of people; tangible actions guided to their physical possessions; intangible actions directed to the minds of people and intangible acts focused on their intangible assets.

**Service quality models**

The Spanish Association for Quality (2017) defines quality models as the source or reference applied by companies to improve their results; they are guidelines that guide, they are not standards to be met. In Duque's opinion (2005), when the concepts of quality are examined, two critical branches can be identified: objective quality and subjective quality, the former being oriented to the producer's perspective and the latter to the consumer's perspective. It is this customer perspective that generates a series of models focused on the evaluation of service quality, in which the customer's point of view is what defines the quality of the service. Duque (2005) describes that the most widely cited measurement models that have emerged from this differentiation between product quality and service quality are the following:

The model of the image of Grönroos recognized as the Nordic school; the SERVQUAL model of Parasuraman, Zeithaml and Berry known as the American school, also clarifying that up to now it is the most used for academic studies; the model of the three components of Rust and Oliver based on what Grönroos proposed and the SERVPERF model of Cronin and Taylor in which they formulated a more detailed scale than SERVQUAL and named it SERVPERF.

**Image model or duvet.**

According to Torres and Vásquez (2015), Christian Grönroos, who is the founder of the school of service management and marketing thought, commonly known as the Nordic School of Marketing, proposes a model of service quality that is composed of three elements: the first, technical quality, which explains the "what" of the service received by the customer, represents the objective characteristic. The second, functional quality, represents "how" the customer obtains and experiences the service, which describes how the service has been delivered. And finally, the corporate image indicates the result of how the customer perceives the organization for the service it provides.

**Model of the three components**

As described by Vargas and Aldana (2006), the three-component model presented by Roland Rust and Richard Oliver in 1994 is based on the Nordic school proposal and is composed of three elements: first, the service and its characterization; second, the delivery process; and third, the environment surrounding the service. Similarly, Duque (2005), explains that the service and characterization, should be considered the design of the service before being delivered to the end-user, the specifications of that design are based on the target market, i.e. customer expectations. Therefore, in this first element, the importance of knowing, on the part of the service producer, the parameters that the final consumer expects is highlighted. The second element of this model is the delivery process and Vargas and Aldana (2006), emphasize that special care must be taken in the requirements that the client has established for the key moment of the reception of the service, where Grönroos (1994), describes it as the moment of truth.

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And finally, the third element, which refers to the environment where the service is developed, Vargas and Aldana (2006), comment on the existence of two perspectives, the internal and the external one. The first points to the environment within the organization where the service is designed, i.e. the structure of the entity; and the external one refers to the way in which the customer looks at the producer of the service, such as its facilities and its prestige in the field.

**SERVPERF model**

The SERVPERF model, derived from the term Service Performance, is described by Torres and Vásquez (2015), as an alternative model to the SERVQUAL model. Cronin and Taylor developed it in 1992 and proposed to evaluate the quality of service based solely on the client's perception. On the same model, Vargas and Aldana (2006), add that its creators use the same 22 statements of the SERVQUAL model related to the five dimensions such as tangibility, reliability, responsiveness, security and empathy.

They further explain that this model also uses the seven-point Likert scale, where they assign one to the statement that is in total disagreement with the customer's perception and seven to the statement that is in total agreement.

Torres and Vásquez (2015) confirm that the SERVPERF model eliminates the use of customer expectations when evaluating service quality. They explain that this is because there is not enough evidence to confirm that the user uses the expectation as a reference point and also because of the tendency of users to exaggerate expectations.

**Generalities of the SERVQUAL service quality model**

Concerning customer expectations of service, and according to the SERVQUAL model, Matsumoto (2014) defines expectation as the belief in the reception of the service and adds that expectation is used as the point of comparison to evaluate service quality. Also, Matsumoto (2014), summarizes it in a straightforward phrase: "what the customer expects from a service" (p. 185).

Zeithaml, Parasuraman, and Berry (1990) add that four factors can influence the expectations of service customers: first, the opinions of the people who form their influence group; second, the individual needs of the customers themselves; third, the experiences they have had with the service; and finally, the communication they receive from the service producers themselves.

As indicated by Zeithaml and Bitner (2002), perception in the SERVQUAL model is the action of how the client values the service at the time of receiving it. Similarly, Brady and Cronin Jr. (2001), explain that perception is made at the moment the service is delivered. Thus, perception for the SERVQUAL model is the appreciation of the qualities that the customer gives to the service received. On the other hand, the SERVQUAL model developed by Zeithaml, Parasuraman and Berry (1990), deduces that there are five criteria or dimensions on which customers perceive the service and defines them as follows: One, tangible elements, are the physical aspects of the service producer, such as equipment and infrastructure; two, reliability, is taking for granted that the service will be performed according to the established standards; dimension three, responsiveness, refers to the promptness of response and the ability and willingness to serve; four, security, is the confidence that the client perceives in the organization providing the service; and dimension five, empathy, is when the service is individually tailored to the client's taste.

The creators of the gap model, Zeithaml, Parasuraman, and Berry (1990), explain that the causes of dissatisfaction in service quality can be identified as five deficiencies or discrepancies and explain them as follows:

Deficiency 1. It is the discrepancy between user expectations and the perceptions of managers and refers to when managers are unaware of customer expectations and focus their efforts in other directions.

Deficiency 2. It is the discrepancy between management's perceptions and specifications or quality standards, arises when managers are aware of customer expectations, but due to certain circumstances, cannot land them in standardized processes within the organization.

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Deficiency 3. The discrepancy between quality specifications and service delivery, this gap arises when, despite process specifications being aligned with client expectations, the staff providing the service to the client is not trained and does not have the resources to perform the service as required.

Deficiency 4. The difference between service delivery and external communication, the discrepancy occurs when the promises of those responsible for delivering the advertising to the clients exceed the actual service perceived by the client.

Deficiency 5. Also known as the overall deficiency, it refers to the difference between customers' expectations and their perceptions of the service received.

In sum, Zeithaml and Bitner (2002), explain that the main focus of this model is the discrepancy between customer expectations and their perceptions, but to close this gap, the model proposes that the first four deficiencies described above, which are known as the service provider gaps, must be closed. As a result of the above, it can be stated that the internal discrepancies of the service producer are the causes of the negative difference between the client's expectations and his or her perception, which causes his or her dissatisfaction and in turn, leads to poor service quality.

## Method

The subject is a public institution in the state of Sonora, which provides undergraduate and graduate-level education, the purpose of this research is to measure the quality of educational service from the point of view of the students of the career an educational program. Therefore, the population is the students enrolled in that degree in the January-May 2018 semester; the number of students on campus is 263.

The sample will be non-probabilistic by quota, as explained by Hernández at al. (2014), the sample by quota is commonly used in opinion studies, quotas are formed based on the proportion of some variable of the population; for this study, it is vital to be able to identify if the perception of the quality of the service is different according to the semester that the student is taking, that is why the groups are divided by semester.

For the moment of the development of the investigation the following semesters are counted: second, fourth, sixth and eighth and higher. Therefore, the sampling will be stratified proportionately; it was determined by calculating the proportion of each group in relation to the total population. Subsequently, this proportion of each group is multiplied by the sample size, and with this, the sample size is obtained. The results are shown in Table 1.

| Group | Number of students | Proportion | Sample size |
|---|---|---|---|
| Second semester | 61 | 23.19% | 36 |
| Fourth semester | 70 | 26.62% | 42 |
| Sixth semester | 62 | 23.57% | 37 |
| Eighth and largest semester | 70 | 26.62% | 41 |
| Totals | 263 | 100.00% | 156 |

**Table 1** Sample stratification
*Source: Prepared by the authors*

The instrument used to evaluate the quality of the service will be the questionnaire developed by Mejías (2005) made up of 22 multiple choice and statement evaluation reagents using the Likert scale that is between one and seven, which was called SERVQUALing. Likewise, we include a section in which the respondent assigns one hundred points among the five dimensions. Also, we include another section of complementary questions.

In this research, twenty-two items are established and classified into five dimensions, which are Tangible Aspects, Reliability, Responsiveness, Security, Empathy.

Procedure

Firstly, the variables that were studied were determined, and the instrument to collect information was established. For this reason, we used the questionnaire already tested by Mejías (2005), generated in his proposal of the SERVQUALing model. Subsequently, we determined the population to which the study is directed, we calculated the sample size and stratified to observe the particular behaviours of the different identified population groups. The next step was the application of the instrument to the sample; the surveys will be sent to the students via email.

With the application of Google Forms, the answers will be automatically received at the source email address. The next step is to present the results obtained, exposing them in the breakdown; the discussion and comments of the results will continue, and at the end of this step we will interpret the results, comparing them with the results of other studies similar to it.

## Dimensions that students consider most important when evaluating service quality

As can be seen in table 2, most second semester students consider the dimensions of tangible aspects and reliability important; regular, the dimension of responsiveness; unimportant, the dimension of security; and unimportant, the dimension of empathy. The majority of fourth, sixth and eighth semester students consider the tangible aspects dimension to be significant, the reliability dimension to be necessary, the response capacity dimension to be regular, the security dimension to be unimportant and the empathy dimension to be unimportant.

Overall, 43% of students consider very importantly the number one characteristic, the appearance of physical facilities, equipment, personnel and communication material used by a Higher Education Institution, which corresponds to the dimension of tangible aspects. 39% of the students consider important the number two characteristic, the ability of a Higher Education Institution to perform the promised service in a safe and precise manner, which corresponds to the dimension of reliability.

Similarly, 47% of the students consider the number three characteristic, the willingness of a Higher Education Institution to help customers and give them a fast service, to be regular, which corresponds to the responsiveness dimension. Likewise, 34% of the students consider unimportant the characteristic of number four, knowledge and friendly treatment of the employees of an Educational Institution of Higher Level and their ability to transmit a feeling of faith and confidence, which corresponds to the dimension of security. And finally, 53% of the students consider unimportant the number five characteristic, care, individualized attention that an Educational Institution of Superior Level gives to its clients, which corresponds to the dimension of empathy.

| Dimension | Characteristic | 2nd semester | | 4th semester | | 6th semester | | 8th semester | | GLOBAL | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | students | % | students | % | students | % | students | % | students | % |
| Tangible aspects | 1. Appearance of physical facilities, equipment, personnel and communication materials used by a Higher Education Institution | | | | | | | | | | |
| | 5. Very important | 13 | 36% | 18 | 43% | 14 | 38% | 22 | 54% | 67 | 43% |
| | 4. Important | 15 | 42% | 7 | 17% | 5 | 14% | 6 | 15% | 33 | 21% |
| Reliability | 2. Ability of a Higher Education Institution to perform the promised service safely and accurately | | | | | | | | | | |
| | 4. Important | 12 | 33% | 16 | 38% | 13 | 35% | 20 | 49% | 61 | 39% |
| Responsiveness | 3. Provision of a Higher Level Educational Institution to assist clients and provide them with prompt service | | | | | | | | | | |
| | 3. Regular | 18 | 50% | 18 | 38% | 16 | 43% | 19 | 46% | 71 | 47% |
| Security | 4. Knowledge and friendly treatment of the employees of a Higher Education Institution and their ability to transmit a feeling of faith and trust | | | | | | | | | | |
| | 5. Very important | 9 | 25% | 8 | 19% | 10 | 27% | 9 | 22% | 36 | 23% |
| | 2. Low importance | 15 | 42% | 15 | 36% | 10 | 27% | 13 | 32% | 53 | 34% |
| Empathy | 5. Care, individualized attention that a Higher Education Institution gives to its clients | | | | | | | | | | |
| | 1. Not important | 22 | 61% | 22 | 52% | 19 | 51% | 19 | 46% | 82 | 53% |

**Table 2** Ranking of the importance of the characteristics of the service, from the student's point of view. Highest values obtained
*Source. Prepared by the authors*

## Validation of the classification of the importance of dimensions

To validate the information obtained in the question about the importance that students attribute to the characteristics, three additional questions are applied. In the first question, it is asked which of the five characteristics is the most important, in the second question, which is the second most important, and in the last question, which is the least important characteristic. The results of these three questions are shown in Table 5.

| Dimension | 2nd semester | | 4th semester | | 6th semester | | 8th semester | | GLOBAL | |
|---|---|---|---|---|---|---|---|---|---|---|
| | students | % | students | % | students | % | students | % | students | % |
| **Tangible aspects** | 8 | 22% | 16 | **38%** | 13 | **35%** | 19 | **46%** | 56 | **36%** |
| **Security** | 12 | **33%** | 15 | 36% | 9 | 24% | 8 | 20% | 44 | 28% |
| **Tangible aspects** | 10 | **28%** | 9 | 21% | 4 | 11% | 6 | 15% | 29 | 19% |
| **Reliability** | 7 | 19% | 14 | **33%** | 11 | 30% | 11 | 27% | 43 | **28%** |
| **Security** | 5 | 14% | 8 | 19% | 12 | **32%** | 13 | **32%** | 38 | 24% |
| **Tangible aspects** | 11 | **31%** | 11.00 | **26%** | 7 | 19% | 4 | 10% | 33 | 21% |
| **Empathy** | 9 | 25% | 6.00 | 14% | 19 | **51%** | 15 | **37%** | 49 | **31%** |

**Table 3** Validation of the importance of the characteristics of the service, from the student's point of view. Highest values obtained
*Source: Prepared by the authors*

As can be seen in Table 3, the group of second semester students specify that characteristic number four, knowledge and friendly treatment of the employees of an Educational Institution of Higher Level and their ability to transmit a feeling of faith and confidence, is the most important. In contrast, the fourth, sixth and eighth-semester groups indicate that the most essential characteristic is the number one, appearance of the physical facilities, equipment, personnel and communication material used by an Institution of Higher Education.

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And seen globally, with 36%, the most important characteristic is number one. Therefore, confirming the result obtained in the previous section, students consider as the most essential characteristic the tangible aspects, that is, the appearance of the physical facilities, equipment, personnel and communication material used by a Higher Education Institution.

On the characteristic as the second most important, the second-semester students selected the dimension of tangible aspects.

The fourth and sixth-semester students chose dimension number four, security, which refers to knowledge and friendly treatment of the employees of an Educational Institution of Higher Level and their ability to transmit a feeling of faith and confidence. The eighth-semester students determined that the second most important characteristic is number two, the ability of a Higher Education Institution to perform the promised service safely and accurately. Overall, the second most important characteristic is number two, the ability of an Institution of Higher Education to perform the promised service safely and accurately. Therefore, the result achieved in the previous section, where the reliability dimension is in second place, is also confirmed.

Concerning the less critical characteristic, the second and fourth-semester students chose the characteristic of appearance of the physical facilities, equipment, personnel and communication material used by an Educational Institution of Higher Level, on the contrary, the sixth and eighth-semester students chose the characteristic that corresponds to the empathy, care, individualized attention that an Educational Institution of Higher Level gives to its clients.

Overall, empathy was selected as the least important characteristic. With them, in the same way, the result obtained in the question about the classification of importance is confirmed, where empathy, that refers to the care, individualized attention that an Educational Institution of Superior Level gives to its clients, is catalogued as nothing important.

Considering the groups in which the sample under study was segmented, the rating given to the service from lowest to highest is as follows: eighth semester 4.88, good; sixth semester 5.11, good; second semester 5.36, very good; and fourth semester 5.38, very good. In general, students in the Industrial Engineering career, qualify the service of this institution with a score of 5.18, which is equivalent to good. The above can be seen in a graphic in Figure 1.

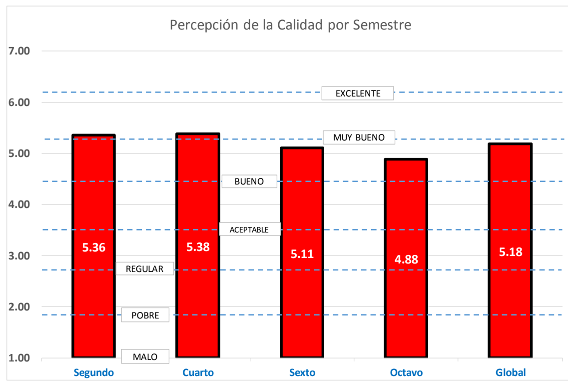


**Figure 1** Perception of service quality by semester
*Source: Prepared by the authors*

Similarly, the variable with the fewest points obtained is one: The Institution has modern-looking machinery and equipment, with 3.80 points. On the other hand, the highest points were obtained by variable fifteen: the student feels safe in his transactions with the institution, with 5.72 points, which corresponds to very good.

As a discussion of the results, we highlight three points: First, the order of importance that the students of the major attribute to the characteristics of the service which is the following: as very important they consider dimension one, tangible aspects; as important, dimension two, reliability; as regular, dimension three, response capacity; as not very important, dimension four, security; and not important, dimension five, empathy. This classification of the importance of service characteristics refers to customer expectations, as presented in the theoretical framework.

Secondly, we present the results of the quality rating: the highest rated dimension is dimension four, security; the second-highest rated is dimension three, responsiveness; the third highest-rated is dimension five, empathy; the fourth highest-rated is dimension two, reliability.

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And the lowest rated is dimension one, intangible aspects. If we place this point with those seen in the theoretical framework, this rating refers to the customer's perception of the service he receives. These results can be seen in Table 4.

| | Dimension | Feature | Expectation | Perception | Rating |
|---|---|---|---|---|---|
| 1 | Tangible aspects | 1. Appearance of the facilities | 5. Very important | 4.59 | Good |
| 2 | Reliability | 2. Ability of a Higher Level Educational Institution to perform the service | 4. Important | 5.09 | Good |
| 3 | Responsiveness | 3. Willingness of an Institution to help clients and give them a fast service | 3. Regular | 5.35 | Very Good |
| 4 | Security | 4. Knowledge and friendly treatment of the employees of an institution | 2. Low importance | 5.58 | Very Good |
| 5 | Empathy | 5. Care, individual attention | 1. Not important | 5.29 | Good |

**Table 8** Results of the expectations and perceptions of the students under study
Source*: Prepared by the authors*

Finally, it is relevant to indicate that the final grade, which students assign to ITESCA's quality of service is 5.18; which is interpreted as good service. If we consider that 7.0 is the maximum score according to the scale used, the percentage obtained is 74%.

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# Humidity and comparative analysis of durability index in pellet of balanced foods for birds

# Análisis de humedad y comparativo de índice de durabilidad en pellet de los alimentos balanceados para aves

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**Abstract**

The research was carried out to improve the humidity parameters and durability index in the balanced feed for poultry in flour and pellet, to guarantee the quality and avoid contamination in the mycotoxin finished product. Samples of flours and pellets were used for four months to evaluate and standardize the pelletization process, to produce balanced food, which is intended for bird consumption in each of the breeding, posture and fattening phases, guaranteeing them a healthy development. In this investigation, we worked with two pelletizing machines of model 700 and model 702 to keep a statistical control of humidity and durability index (IDP), to avoid product losses in the pelletizing process. The contribution of the present investigation was to reduce the economic impact of the feed processing plant since reprocesses were significantly reduced.

**Quality, Humidity, Durability**

**Resumen**

La investigación se realizó para mejorar los parámetros de humedad e índice de durabilidad en los alimentos balanceados para aves en harina y pellet, para garantizar la calidad y evitar la contaminación en el producto terminado por micotoxinas. Durante cuatro meses se utilizaron muestras de harinas y pellet con el objetivo de evaluar y estandarizar el proceso de peletizacion, para producir alimento balanceado, el cual está destinado para consumo de aves en cada una de las fases de crianza, postura y engorda, garantizándoles un sano desarrollo. En esta investigación se trabajó con dos máquinas peletizadoras de modelo 700 y modelo 702 para llevar un control estadístico de humedad e índice de durabilidad (IDP), para evitar pérdidas de producto en el proceso de peletizado. La contribución de la presente investigación fue reducir el impacto económico de la planta procesadora de alimentos balanceados, ya que se disminuyeron significativamente los reprocesos.

**Calidad, Humedad, Durabilidad**

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## Introduction

Pelletization consists of the agglomeration of small particles of a mixture into long units or dense tablets employing a mechanical process combined with moisture, heat, and pressure, all of which determines an improvement of the characteristics in livestock feeds (Benhnke K, 2001). Nutritionally, pelletization allows a natural increase of liquid energy in diets, due to the gelatinization of carbohydrates, reducing energy expenditure in the apprehension of food (McKinney and Teeter, 2004).

The production of quality balanced foods begins with the selection of quality ingredients. The two most common forms of balanced poultry feeds are pellets and flour.

The pelleting process increases the cost of processing, but this cost is recovered by improving some aspects of the feed, the feed conversion improves by 6% to 7% compared to flour.

## Justification

To guarantee the quality and safety of the balanced food, controlling the humidity of the flours and the finished product to control the durability index of the pellet using a statistical control of humidity and durability index to standardize the processes and avoid economic losses.

The analysis and control of humidity in model 700 and model 702 machines will allow obtaining better pellet formation that will contribute to a better quality of the finished product as well as in the nutritional quality when conglomerating all the nutrients that the birds must consume.

## Problem

The loss of moisture in some part of the process makes the formation of pellets is not carried out, this being a relevant factor that generates an economic impact of the food processing plant because when obtaining reprocessing money is lost, as well as ensuring the quality and safety of products produced for customer satisfaction.

## Hypothesis

The exact determination of moisture and the product shelf life index (PDI) in poultry feeds is favorable for obtaining a contamination-free feed of acceptable quality.

## Objectives

### Overall objective

Moisture determination and durability index in poultry feeds, guaranteeing better quality, avoiding contamination in the finished product of the feed processing plant.

### Specific Objectives

- Determine moisture control by standardizing the raw material process.
- Monitor the humidity percentage with the thermobalance for the mixing process of the flours and the finished product.
- Temperature verification for humidity control
- Record and graph the data obtained for the control of humidity and the Durability Index in products. (PDI)

## Theoretical framework

### The pellet in the poultry industry

Physiologically, the process of apprehending the bird's food is given by two elements: its vision as the best within the group of vertebrates; and its beak as a keratinized structure, irrigated and innervated by a nervous membrane coming from the trigeminal nerve; this membrane has the so-called "mechanoreceptors," which give the bird the capacity to select certain food particles by means of constant pecking of the food (López C. 2005).

The pelletization process results in faster growth and generally a lower feed conversion rate. The reason for this improved performance is the energy released by the pressing of the mixture. Birds and generally other (production) animal species fed pelleted feed eat about the same number of meals per day as those fed meal but spend less time consuming the pellets (Jansen, 2001 cited by Minoru Miyasaka, 2004).

The transit of the feed through the gastrointestinal tract is regulated primarily by the crop, where the granulometry plays an important role: fine feeds determine a longer filling time and shorter retention time due to the ease of lubrication and homogenization of the chyme within the crop. Therefore the transit at the intestinal level will be faster, and the absorption process will not be the best; but in granulated feeds, the crop filling time will be shorter, the retention time longer, and the transit of the feed more efficient in terms of absorption and enzymatic activity.

The pellet compared to flour improves the performance of broilers, because more than favoring the development of the gastrointestinal tract, increases the digestibility of nutrients, reducing the viscosity of intestinal content and positively modifies the microfiber. The increase in the digestibility of carbohydrates is due to the breakdown and cooking of granules containing amylase and amylopectin, which facilitates enzymatic action at the digestive level with more contact time and better production. The pelletization when solubilizing partially to the proteins by the alteration of its natural structures releases several nutrients with the rupture of the cellular walls (López C.C.,1999)

**Durability**

Durability is the number of fines produced from a sample of pellets after they have been subjected to mechanical or pneumatic agitation. (Thomas & Van der Poel 1996)

The term durability is defined as a pattern or parameter of physical analysis and is based on the number of pellets or crumblers recovered after being subjected to mechanical or pneumatic agitation, which simulates the transport and handling of pelleted feed under normal conditions within the plant. This parameter is expressed as a percentage.

An IDP with a percentage of (92-95%) qualifies a high-quality, durable feed.

(Aarseth et al. 2006b) studied the effect of extrusion temperature on pellet tensile strength. The extrusion temperatures used in their study were 100°C and 140°C. They observed that pellets extruded at higher temperatures showed lower strength, although they were more homogeneous.

Pellet formation occurs at the point where the rollers and the die or output die come into contact. All other activities, such as conditioning, cooling, etc., support the point of contact (Behnke, 2010).

The final moisture content depends on the initial moisture content of the raw materials, although during the production process, we can lose or gain moisture in each of the different production phases. (Stevens 1987) found that as the conditioning temperature increases, the degree of gelatinization of starches decreases. Poultry feeds are conditioned in a temperature range between 80 to 85 °C, and a steam pressure of 138 Kpa and 552 Kpa, with a conditioning time between 30 and 60 seconds.

**Methodology**

The present investigation was carried out in a poultry feed processing plant, over a period of six months. It consists of three research stages:

**Stage 1 Data collection and analysis:** During data collection, the process was analyzed in both morning and afternoon shifts at the poultry processing plant, in order to identify flaws in the pellet moisture and durability index parameters. Moisture analyses of raw materials and finished products were performed in the plant's physical analysis laboratory area on a daily basis for four consecutive months to perform statistical analysis.

**Stage 2 Process Standardization:** A process flow diagram was made Moisture analysis, which consists of the following steps:

– Raw material reception
– Analysis of temperature, humidity, weight/volume of the raw material
– Mixing of raw materials
– The flours go through a thermo-conditioner, where they are mixed with 80% steam
– Peel the flours on a die.
– The product goes through a cooler for 10 minutes.
– It goes through a grinder, depending on the presentation of the food.
– It arrives in the sieve, where it is in charge of separating the flours from the thick

- The pellet samples are analyzed: the sample is taken from the pellet to make the corresponding moisture and durability index analyses of the pellet.
- It passes through an elevator to reach the hoppers for packaging
- Packed in 20 or 40-kilogram presentations
- Storage
- Marketing.

**Stage 3:**

Processing of the information obtained for statistical analysis in order to determine the PDI, as well as the standardization of humidity for pellet quality control.

## Results

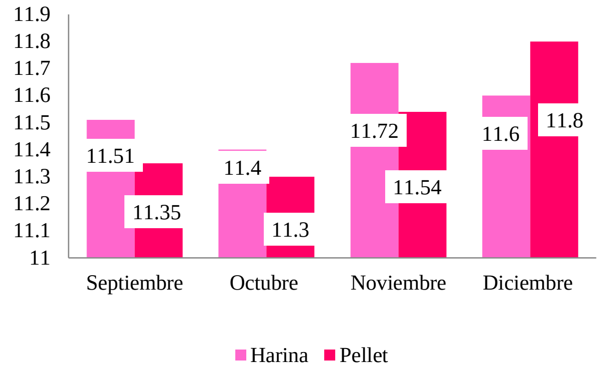
The right moisture level is important because it reduces energy consumption during the compression (pelletizing) process and ensures that production flows smoothly and reduces the risk of blockages. This also serves to reduce damage caused by excessive heat. It also ensures better pellet quality as the optimum level positively affects the hardness of the pellet.

The negative side of increasing the humidity would be to do so with "free" water, which puts the quality of the feed at risk because it would be available for the development of undesirable microorganisms creating rapid growth of fungi with the risk of mycotoxin production.

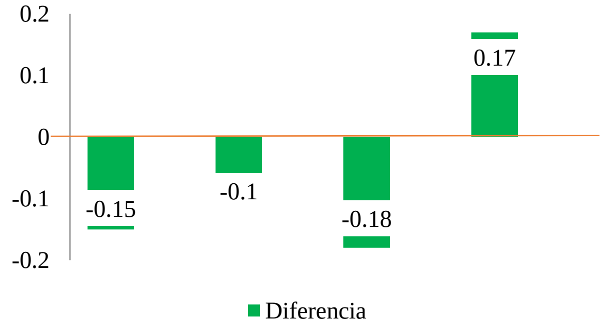
This typically results in a 0.5 to 1% loss or depletion in the finished feed compared to the raw materials. Moisture compensation is performed during the mixing process; practical experience shows that adding water alone has a limited potential: after cooling the moisture level in the finished feed is at most equal to the moisture level when it enters the press, even with high additions of water alone, if free water is added the risk of fungal growth is increased, see table 1.

| Months | Humidity | | Difference |
|---|---|---|---|
| | Flour | Pellet | |
| September | 11.51 | 11.35 | -0.15 |
| October | 11.4 | 11.3 | -0.1 |
| November | 11.72 | 11.54 | -0.18 |
| December | 11.6 | 11.8 | 0.17 |

**Table 1** Final Results of Humidity
*Source Own elaboration*




**Graph 1** Results of humidity
*Source: Prepared by the authors*




**Graph 2** Humidity Difference
*Source: Prepared by the authors*

Debido a estas pérdidas los productores buscan aprovechar los beneficios de eficiencia y calidad que les da el nivel óptimo de humedad en el alimento terminado.

## Durability Index

Precise measurement of the durability of the pellets allows for an optimal incorporation rate of the binders. Too little pellet binder causes pellet breakage; too much is a waste of money. Lignin, molasses, starch, and steam are the most popular binders.

Today the highest cost is the amount of energy. Granulation is an energy-intensive process. Pelleting presses operate under pressure and are very energy intensive. The goal is to minimize the energy cost and maximize the efficiency of the pelletizing process.

To calculate the Pellet Durability Index, we divide the weight of whole pellets by 500 and multiplying it by 100. The following Table shows the results of PDI calculation for the two models of machines.

| Maq | Month | Code | PDI % |
|---|---|---|---|
| **700** | **September** | **290** | **29** |
| 700 | October | 290 | 40 |
| **702** | **September** | **290** | **32.5** |
| 702 | October | 290 | 53 |
| 700 | November | 290 | 40 |
| 702 | November | 290 | 61 |
| 700 | December | 290 | 40 |
| 702 | December | 290 | 67 |

**Table 2** Durability Index Results

The comparison of PDI% allowed us to determine the hardness of the pellet, indicating the quality of the finished product, concluding that the lower data (29 and 32.5) are the data that gave us the best PDI.

## Conclusions

In this work, we conclude that the obtaining of the humidity of flours as a finished product was registered, and we graphed the results obtained in the drying with the thermobalance. In the graph the results obtained during four months in the first month a flour of 11.51 a finished product of 11 was obtained.35 and a difference which means that in drying a moisture loss of -0.15 was obtained, in the second month a flour of 11.40 was obtained, a finished product of 11.30 and a difference of -0.1, in the third month a flour of 11.72 was obtained, a finished product of 11.34 and a difference of -0.18, in the last month a flour of 11.60 was obtained, a finished product of 11.80 and a difference of 0.17. We found that the moisture content also varies depending on the product to be pelletized.

The humidity of the finished product cannot exceed an optimum of 12%. Therefore, we conclude that the humidity of such products was kept at the standard so as not to affect the shelf life.

If a product exceeds the optimum humidity, this causes reprocessing and economic and time losses to the company. Thus, this project resulted in more favorable humidity control.

In the verification of temperatures and tons worked per hour of the 700 and 702 pelletizers for their performance. We kept a daily record that allowed a specific control of how much production is worked per shift, as that depends on having a good durability index.

## Acknowledgments

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# Analytical proofs of expected value and variance of the main functions of continuous probability distributions, with examples of their applications

# Demostraciones analíticas de valor esperado y varianza de las principales funciones de distribuciones de probabilidad continuas, con ejemplos de sus aplicaciones

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**Abstract**

The present work proposes the design of mathematical problems, which allow the adequate understanding of mathematical concepts of probability for its correct interpretation and later application in the resolution of probabilistic problems. For the development of this work we rely on the theory of didactic situations of Brousseau (1997) and Sadovsky (2005). We believe that new materials and didactic models of this type have great educational potential because they encourage the analysis and understanding of various probability problems (Panizza, 2003). Accurate communication between teachers and students in the approach, interpretation, resolution, and testing of probability problems is of vital importance. The software used for this purpose is the MATHEMATICA program, a tool that facilitates the writing of formulas and calculations, as well as the construction of graphs, through a friendly interface, facilitating the self-taught work of the student and encouraging the development of analysis skills and problem solving. We believe that these materials will contribute to the teaching and learning processes of probability at higher education levels.

**Continuous Probability Distributions, Expected Value, Variance**

**Resumen**

El presente trabajo propone el diseño de problemas matemáticos, que permitan la adecuada comprensión de conceptos matemáticos de probabilidad para su correcta interpretación y posterior aplicación en la resolución de problemas probabilísticos. Para el desarrollo de este trabajo nos apoyamos en la teoría de situaciones didácticas de Brousseau (1997) y Sadovsky (2005). Creemos que nuevos materiales y modelos didácticos de este tipo tienen un gran potencial educativo porque fomentan el análisis y comprensión de diversos problemas de probabilidad (Panizza, 2003). Es de vital importancia la comunicación precisa entre los profesores y estudiantes en el planteamiento, interpretación, resolución, y comprobación de problemas de probabilidad. El software utilizado para este propósito es el programa MATHEMATICA, herramienta que facilita la escritura de fórmulas y los cálculos, así como la construcción de gráficos, mediante una interfaz amigable, facilitando el trabajo autodidacta del estudiante y fomentando a la vez el desarrollo de las habilidades de análisis y resolución de problemas. Consideramos que estos materiales permitirán coadyuvar en los procesos de enseñanza y aprendizaje de la probabilidad en el nivel de educación superior

**Distribuciones de probabilidad continuas, Valor esperado, Varianza**

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## Introduction

This paper focuses on the proofs of expected value and variance of the main continuous probability distributions by applying algebraic procedures supported by artifacts. The concepts involved are developed to understand them in a natural, inductive, and deductive way. The teaching of probability and statistics has presented a significant development in the last years due to its increasing application in several fields of science, technology, and social and administrative disciplines.

Many institutions and educational centers in the world devote great efforts to designing and updating specific curricula and materials. Difficulties have been detected in the teaching-learning process of probability and its impact on the quality of education.

For this proposal, we rely on two approaches due to the nature of probability and its application, the first one referring to the Theory of Didactic Situations Brousseau (1997); Sadovsky (2005). Due to its high educational potential and is relevant to this proposal since it seeks the conditions in an environment and analysis of mathematical knowledge, under the hypothesis that they are not built spontaneously (Panizza, 2003). It is based on the premise that knowledge is not transmitted from one person to another, but that the individual constructs his or her knowledge. The role of the teacher is the key to the rigorous design and judicious and appropriate choice of problems.

The second approach referring to the theory of constructivist knowledge guiding this work, under this strategy, we have chosen the problem-solving method, since it allows interaction in concrete and significant situations, stimulating knowledge, know-how and know-how-to-be, that is, the conceptual, procedural and attitudinal aspects. Thus, students are in a permanent process of acquiring knowledge in all their contexts, since, for constructivism, the most important thing is not the new knowledge itself, but acquiring new skills with it, which allows students to apply what they already know to a new situation. Specifically, we will address algebraic procedures in the proofs of concepts of expected value and variance.

## Justification

Several reasons justify the proposal of this work, the first one is concerned with the teaching work: it is known that the probability topics are of the most complex for the students of all the educational levels; this is also reported by the literature in Statistical Education (Batanero y Godino, 1997). The second is that it is present from the basic level in the probability curriculum in the Mexican education system.

However, students finish their instruction without having acquired the knowledge and understanding of the concepts. Many times certain subjects are not revised, even within curricular guidelines, and when they are, it is done in a procedural rather than conceptual manner. The third is that, just as students have difficulty understanding concepts of probability, teachers have great difficulty in teaching them comprehensively and clearly.

## Problem

This problem is associated with several factors: since there is little research in this field to guide teachers. According to the teaching experience and works presented in different national and international forums, there are not enough didactic resources to support teaching; the textbooks used to teach probability and statistics give more importance to the procedure than to the understanding, and the exploratory approach is reduced.

It is worth mentioning the books by William Mendenhall (2013), M. Mood Alexander (1978); Ronald E. Walpole (2012) only mention functions and their respective parameters but omit detailed proofs, but in the case of George C. Canavos (1988) present some incomplete developments.

Another important aspect is the scarce national bibliography of probability texts in the last years since they are mainly published in their original language such as English, French, German, Russian, Italian, etc. under the environment of their own cultural values and educational systems, from which many of them have been translated and used as basic texts in probability courses here in Mexico, lacking an own identity, so it is important to write texts under our cultural environment.

## Hypothesis

Competences are created for the solution of mathematical problems of probability by adequately selecting examples that allow for self-learning in the student, which allows for the development of autonomy and security to undertake more complex activities.

## Objectives

To develop in detail the proofs of expected value and variance of the main continuous probability distributions using algebraic procedures supported by mathematical artifices and integration techniques.

## Specific Objectives

That this material is a guide that helps to strengthen and develop capacities and skills, as well as the autonomous character of the student in the process of his or her professional training at a higher level. These include the following:

- Expressing ideas clearly
- Structuring ideas in a logical way
- Structure graphs, tables and diagrams that help to obtain the desired result.
- Use appropriate mathematical language and representation
- Selection of appropriate mathematical tools (ICTS)
- Demonstrate knowledge and understanding.
- Apply mathematics in different contexts
- Apply problem-solving techniques
- Recognize and explain patterns, where appropriate
- Generalize and justify conclusions

## Theoretical framework

In this research, where solving analytical problems is a challenge, it is also an opportunity for the student to develop and acquire skills in solving mathematical problems. It is essential to make clear that the development of these skills is the result of personal work, of the practice acquired by solving problems and of reflection on that practice. It is not possible to become a skilled problem solver by merely reading a book, just as it is not possible to become a good sportsman or pianist by merely reading.

However, the knowledge of the appropriate techniques and of the typical mistakes to be avoided can be as useful for the solver as it is for the sportsman or concert musician. It is clear that, in any of the mathematical branches or knowledge, there inevitably arises the part of concepts and practical application, which is usually called "problem," and which varies as time, place, or circumstances go by.

On the other hand, it is known that problem-solving promotes learning, which is why its inclusion in curricula is important, constituting almost an autonomous discipline within mathematical education. Problem-solving is considered essential in mathematics education. Through problem-solving, students experience the scope and usefulness of mathematics in the real world. Throughout one's life, one will encounter problems that must be faced, both inside and outside the school, since the idea of a problem-free life does not exist.

In a lecture given in 1968, George Polya said: "It is well justified that all mathematical texts contain problems. Problems can even be considered as an essential part of mathematics education". Guzmán M. de (1984) commented that "what we should above all provide our students through mathematics is the possibility of acquiring adequate thinking habits for the resolution of mathematical and non-mathematical problems." What use can a few theorems and properties relating to entities with little meaning be to them in their minds if they are then going to leave them there hermetically sealed? Problem-solving has rightly been called the heart of mathematics, for that is the real value that has attracted and still attracts mathematicians of all time.

From the confrontation with adequate problems motivations, attitudes, habits, ideas for the development of tools can arrive, in a word, the life of mathematics. When we solve problems, we learn fundamentally to understand the functioning of our reasoning, to master our moods, and to increase our self-confidence.

**Research Methodology**

Through the technique of analytical reading and reformulation, which consists of making an in-depth observation and analysis so that the elements and their essential relationships that are presented implicitly or directly are clearly distinguished, allowing an analytical algebraic development by sequentially performing the verification of a mathematical definition, being a guide in the search for ideas for the solution. In general, this technique leads us to develop a mathematical process using techniques and artifices that allow a new structuring of the problem according to a closer language until the desired result is obtained. This is a process of analysis and synthesis.

We can say that research in this area began by being empirical, not systematized. It was interested almost exclusively in standard problems and restricted to quantifications about problem-solving behavior. Currently, however, it uses a wide range of methods (quantitative and qualitative), covers a broad spectrum of problems, and has a theoretical basis.

Campistrous P. L. and Rizo C. (1996), with great objectivity point out: that the methodological procedures are aimed at the actions to be carried out by the teacher focused mainly on teaching methodology, leaving aside the search for procedures for student action, which leads to observe and reflect on significant aspects such as:

1. The stimulation is indirect
2. The generalized forms of performance in the student are not achieved. They are necessary for life.
3. The problems are focused on developing calculation skills and not as an object of teaching.
4. The difficulty of the problems and their parameters are not very precise, which confuses or makes it impossible to build analogies.

Particularly in the problems of mathematical analysis, the meanings are not adequately worked out.

Likewise, they present a series of techniques that allow us to face the challenges of solving the most complex problems of mathematics.

**Type of Research**

We can classify this research as descriptive, characterized by mathematical analysis and the development of techniques and strategies to pose and solve problems where interpretation and verification become relevant in their application.

**Theoretical Methods**

Specifically, we will approach algebraic procedures applying mathematical artifices in the development of the proofs of expected value and variance, to process and write this work and enrich it in its technical character, we rely on the software MATHEMATICA, facilitating the capture and mathematical calculations. In order to obtain the proofs, knowledge of algebra, differential, and integral calculus of a higher level is required.

**Methodology to be Developed**

Definitions and basic concepts of a discrete random variable:

$\boldsymbol{Expected\ value}$:

$$\boldsymbol{E(X) = \int_{-\infty}^{\infty} x f_x(x)\, dx}\text{.......}\textit{(1)}$$

$$\boldsymbol{Var}(\boldsymbol{X}) = \sum_{x=1}^{N} (\boldsymbol{X} - \boldsymbol{E}(\boldsymbol{X}))^2 \boldsymbol{f}(\boldsymbol{X})\text{.....}\textit{(2))}$$

$$\boldsymbol{Var}(\boldsymbol{X}) = \boldsymbol{E}(\boldsymbol{X}^2) - [\boldsymbol{E}(\boldsymbol{X})]^2\text{..........}\textit{(3)}$$

In the following, we present the results of the analytical proofs of expected value and variance of the main continuous probability distributions, which were carried out by resorting to the application of detailed algebraic work and relying on indirect expressions of derivatives and expected value.

**Results**

The developments of the expected value and variance proofs are presented below.

**The random variable X has a Uniform Continuous Distribution if its density function is:**

$$\mathbf{f}(\boldsymbol{x}) = \frac{\mathbf{1}}{\boldsymbol{b}-\boldsymbol{a}},$$

$$\boldsymbol{Con}: -\infty < \boldsymbol{a} < \infty, -\infty < \boldsymbol{b} < \infty$$

*Substituting in (1)*

$E(x) = \int_a^b xf(x)\,dx = \int_a^b x\frac{1}{b-a}dx$
$\frac{1}{b-a}\int_a^b x\,dx$
$\frac{1}{b-a}\int_a^b x\,dx =$

$\frac{1}{b-a}\frac{x^2}{2}|_a^b = \frac{b^2}{2(b-a)} - \frac{a^2}{2(b-a)} = \frac{b^2-a^2}{2(b-a)}$

$$= \frac{(b-a)(b+a)}{2(b-a)} = \frac{b+a}{2}$$

thus: $E(x) = \frac{b+a}{2}$

*Substituting in (1)*

Let: $E(x^2) = \int_a^b x^2 f(x)\,dx =$

$\int_a^b x^2\frac{1}{b-a}dx = \frac{1}{b-a}\int_a^b x^2\,dx =$

$\frac{1}{b-a}\frac{x^3}{3}|_a^b$
$= \frac{1}{b-a}\frac{b^3}{3} - \frac{1}{b-a}\frac{a^3}{3}$
$= \frac{b^3-a^3}{3(b-a)} = \frac{(b-a)(a^2+ab+b^2)}{3(b-a)} = \frac{a^2+ab+b^2}{3}$

*Substituting in 2*

$\mathrm{Var}(x) = E(X^2) - (E(X))^2$
$= \frac{a^2+ab+b^2}{3} - \frac{(b+a)^2}{2^2} =$

$$= \frac{4(a^2+ab+b^2) - 3(a^2+2ab+b^2)}{12}$$

Thus, $\mathrm{Var}(x) = \frac{(a-b)^2}{12}$

The random variable X has a Continuous Normal Distribution if its density function is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, I_{(-\infty,\infty)}(x)$$

With $-\infty < \mu < \infty$, $-\infty < x < \infty$ and $\sigma^2 > 0$

1) Properties:

1.1) It has a bell shape (Gauss).
1.2) It is symmetrical with respect to $x = \mu$.
1.3) It is continuous in: $-\infty < x < \infty$
1.4) The area or probability under the y-curve on the x-axis is equal to 1.
1.5) $P[x \leqslant \mu] = P[x \geqslant \mu] = 1/2$
1.6) $E[x] = \mu$ $\mathrm{Var}(x) = \sigma^2$
1.7) $P[x=a] = 0$

*Substituting in 1*

2)Expected value: $E[x] = \int_{-\infty}^{\infty} xf(x)\,dx =$

$$\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi\sigma^2}}xe^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}dx =$$
$$= \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty} xe^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}dx$$

Changing the variable:

$$y = \frac{x-\mu}{\sigma}$$
$$\Rightarrow x = \sigma y + \mu; \frac{dx}{dy} = \sigma \Rightarrow dx = \sigma\,dy$$

Obtaining the limits of the integral, if: $x = -\infty \Rightarrow y = -\infty$, $x = \infty \Rightarrow y = \infty$

$$\Rightarrow \int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi\sigma^2}}xe^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}dx$$
$$= \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty}(\sigma y + \mu)e^{-\frac{y^2}{2}}\sigma\,dy =$$
$$= \frac{1}{\sqrt{2\pi\sigma}}[\sigma\int_{-\infty}^{\infty} ye^{-\frac{y^2}{2}}dy + \mu\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}}dy]$$
$$= \frac{1}{\sqrt{2\pi}}\sigma\int_{-\infty}^{\infty} ye^{-\frac{y^2}{2}}dy + \frac{1}{\sqrt{2\pi}}\mu\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}}dy$$

In the first integral:

$$u = -\frac{y^2}{2}, \frac{du}{dy} = y, \qquad du = -y\,dy$$
$$\Rightarrow dy = -\frac{du}{y}$$

$$\Rightarrow \frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{y}e^{-\frac{y^2}{2}}dy$$
$$= \frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty} -\mathrm{y}e^{u}\frac{du}{y} = \frac{-\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^u\,du$$
$$= \frac{-\sigma}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}|_{-\infty}^{\infty} = \frac{-\sigma}{\sqrt{2\pi}e^{-\frac{y^2}{2}}}|_{-\infty}^{\infty} =$$
$$\frac{-\sigma}{\sqrt{2\pi}e^{\frac{\infty^2}{2}}} - \left(\frac{-\sigma}{\sqrt{2\pi}e^{\frac{-\infty^2}{2}}}\right) =$$
$$= \frac{-\sigma}{\sqrt{2\pi}(\infty)} + \frac{\sigma}{\sqrt{2\pi}(\infty)} = \frac{-\sigma}{\infty} + \frac{\sigma}{\infty} = 0$$

From the second integral we make change of variable:

$$t = \frac{y^2}{2}, y = \sqrt{2}\sqrt{t}, \text{donde:}$$
$$\frac{dy}{dt} = \frac{\sqrt{2}}{2}t^{\frac{-1}{2}}, \mathrm{dy} = \frac{\sqrt{2}}{2}t^{\frac{-1}{2}}\mathrm{dt}$$

*And because it is a symmetrical integral, it doubles in the interval (0,∞).*

$$\Rightarrow \frac{\mu}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}}dy =$$
$$\frac{2\mu}{\sqrt{2\pi}}\int_0^{\infty} e^{-t}\frac{\sqrt{2}}{2}t^{\frac{-1}{2}}dt = \frac{\mu}{\sqrt{\pi}}\int_0^{\infty} e^{-t}t^{\frac{-1}{2}}dt$$

$$Taking\ to\ a\ Gamma\ function, when$$
$$\Gamma(\frac{1}{2}) = \sqrt{\pi}$$
$$\Rightarrow \frac{\mu}{\sqrt{\pi}}\int_0^{\infty} e^{-t}t^{\frac{-1}{2}}\,dt = \frac{\mu}{\sqrt{\pi}}\int_0^{\infty} e^{-t}t^{\frac{1}{2}-1}\,dt$$
$$= \frac{\mu}{\sqrt{\pi}}\Gamma(\frac{1}{2}) = \frac{\mu}{\sqrt{\pi}}\sqrt{\pi} = \mu$$

**Thus: E[x] = μ**

From ...3. Variance: Var(x) = E[x²] - (E[x])²

$$\mathrm{E}[\mathrm{x}^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$
$$= \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}\,dx$$

Changing the variable:

$$y = \frac{x-\mu}{\sigma}$$
$$\Rightarrow x = \sigma y + \mu; \frac{dx}{dy} = \sigma \Rightarrow dx = \sigma\,dy$$

*Obtaining the limits of the integral, if:* x = - ∞ ⇒ y = - ∞, x = ∞ ⇒ y = ∞

$$\Rightarrow \int_{-\infty}^{\infty} (x)^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2}}\sigma\,dy$$
$$= \int_{-\infty}^{\infty} (\sigma y + \mu)^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2}}\sigma\,dy =$$
$$= \int_{-\infty}^{\infty} (\sigma y + \mu)^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}\,dy$$
$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} (\sigma^2 y^2 + 2\sigma y\mu + \mu^2)e^{-\frac{y^2}{2}}\,dy =$$
$$\frac{1}{\sqrt{2\pi}}[\sigma^2\int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2}}\,dy +$$
$$+2\sigma\mu\int_{-\infty}^{\infty} y e^{-\frac{y^2}{2}}\,dy + \mu^2\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}}\,dy]$$
$$= \frac{\sigma^2}{\sqrt{2\pi}}\int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2}}\,dy +$$
$$+\frac{2\sigma\mu}{\sqrt{2\pi}}\int_{-\infty}^{\infty} y e^{-\frac{y^2}{2}}\,dy + \frac{\mu^2}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}}\,dy =$$

*Making a variable change in the first integral:*

$$= \frac{y^2}{2} \Rightarrow y^2 = 2t, y = \sqrt{2}\sqrt{t} \Rightarrow \frac{dy}{dt}$$
$$= \frac{\sqrt{2}}{2}t^{\frac{-1}{2}}, dy = \frac{\sqrt{2}}{2}t^{\frac{-1}{2}dt}$$

*And because it is a symmetrical integral, it doubles in the interval (0,∞).*

$$\Rightarrow \frac{\sigma^2}{\sqrt{2\pi}}\int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2}}\,dy =$$
$$= \frac{2\sigma^2}{\sqrt{2}\sqrt{\pi}}\int_0^{\infty} 2te^{-t}\frac{\sqrt{2}}{2}t^{\frac{-1}{2}}\,dt =$$
$$= \frac{2\sigma^2}{\sqrt{\pi}}\int_0^{\infty} e^{-t}t^{\frac{1}{2}}\,dt$$
$$Taking\ to\ a\ Gamma\ function, when$$
$$\Gamma(\frac{3}{2}) = \frac{1}{2}\sqrt{\pi}:$$
$$\Rightarrow \frac{2\sigma^2}{\sqrt{\pi}}\int_0^{\infty} t^{\frac{1}{2}}\,dt = \frac{2\sigma^2}{\sqrt{\pi}}\int_0^{\infty} e^{-t}t^{\frac{3}{2}-1}\,dt =$$
$$= \frac{2\sigma^2}{\sqrt{\pi}}\Gamma(\frac{3}{2}) = \frac{2\sigma^2}{\sqrt{\pi}}\frac{1}{2}\sqrt{\pi} = \sigma^2$$
$$Solving\ the\ second\ integral:$$
$$\frac{2\sigma\mu}{\sqrt{2\pi}}\int_{-\infty}^{\infty} y e^{-\frac{y^2}{2}}\,dy$$
$$U = \frac{-y^2}{2}, \frac{du}{dy} = -y, du=$$
$$= -y\,dy \Rightarrow -\frac{du}{y} = dy$$
$$\Rightarrow \frac{2\sigma\mu}{\sqrt{2\pi}}\int_{-\infty}^{\infty} y e^{-\frac{y^2}{2}}\,dy =$$
$$= \frac{2\sigma\mu}{\sqrt{2\pi}}\int_{-\infty}^{\infty} y e^{-u}(-\frac{du}{y})$$
$$= \frac{2\sigma\mu}{\sqrt{2\pi}}\int_{-\infty}^{\infty} - e^{-u}\,du =$$
$$= \frac{-2\sigma\mu}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}\Big/_{-\infty}^{\infty} = \frac{-2\sigma\mu}{\sqrt{2\pi}e^{\frac{y^2}{2}}}\Big/_{-\infty}^{\infty}$$
$$= \frac{-2\sigma\mu}{\sqrt{2\pi}e^{\frac{\infty^2}{2}}} - (\frac{-2\sigma\mu}{\sqrt{2\pi}e^{\frac{(-\infty)^2}{2}}}) =$$
$$= \frac{-2\sigma\mu}{\sqrt{2\pi}e(\infty)} + \frac{2\sigma\mu}{\sqrt{2\pi}e(\infty)} =$$
$$= \frac{-2\sigma\mu}{\infty} + \frac{2\sigma\mu}{\infty} = 0 + 0 = 0$$
$$Making\ a\ variable\ change\ in\ the\ third\ integral:$$
$$\frac{\mu^2}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}}\,dy$$
$$t = \frac{y^2}{2} \Rightarrow y^2 = 2t,$$
$$y = \sqrt{2}\sqrt{t}, \frac{dy}{dt} = \frac{\sqrt{2}}{2}t^{\frac{-1}{2}}, dy = \frac{\sqrt{2}}{2}t^{\frac{-1}{2}dt}$$

*And because it is a symmetrical integral, it doubles in the interval (0,∞)*

$$\Rightarrow \frac{\mu^2}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}}\,dy = \frac{2\mu^2}{\sqrt{2\pi}}\int_0^{\infty} e^{-\frac{y^2}{2}}\,dy$$
$$= \frac{2\mu^2}{\sqrt{2}\sqrt{\pi}}\int_0^{\infty} e^{-t}\frac{\sqrt{2}}{2}t^{\frac{-1}{2}}\,dt$$
$$= \frac{\mu^2}{\sqrt{\pi}}\int_0^{\infty} e^{-t}t^{\frac{1}{2}}\,dt$$
$$Taking\ to\ a\ Gamma\ function, when$$
$$\Gamma(\frac{1}{2}) = \sqrt{\pi}$$
$$\Rightarrow \frac{\mu^2}{\sqrt{\pi}}\int_0^{\infty} e^{-t}t^{\frac{1}{2}}\,dt = \frac{\mu^2}{\sqrt{\pi}}\int_0^{\infty} e^{-t}t^{\frac{1}{2}-1}\,dt$$
$$= \frac{\mu^2}{\sqrt{\pi}}\Gamma(\frac{1}{2}) = \frac{\mu^2}{\sqrt{\pi}}\sqrt{\pi} = \mu^2$$
$$Returning\ to\ the\ main\ integral:$$

$$\int_{-\infty}^{\infty} (x)^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2}} \sigma\, dy$$
$$\int_{-\infty}^{\infty} (x)^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2}} \sigma\, dy$$
$$= \int_{-\infty}^{\infty} (\sigma y + \mu)^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2}} \sigma\, dy =$$
$$= \int_{-\infty}^{\infty} (\sigma y + \mu)^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2}} \sigma\, dy =$$
$$= \int_{-\infty}^{\infty} (\sigma y + \mu)^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy =$$
$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma^2 y^2 + 2\sigma y \mu + \mu^2) e^{-\frac{y^2}{2}} dy =$$
$$= \sigma^2 + 0 + \mu^2, \Rightarrow \mathrm{E}[\mathrm{x}^2] = \sigma^2 + \mu^2$$
$$\mathrm{Var}(x) = \mathrm{E}[\mathrm{x}^2] - (E[x])^2 =$$
$$= \sigma^2 + \mu^2 - \mu^2 = \sigma^2$$
$$\mathrm{Var}(x) = \sigma^2$$

*Density function Gamma, defined:*

$$(x) = \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$$
$$I_{(0,\infty)}(x), \mathrm{con} \lambda > 0 y r > 0$$
$$\text{Where:} \Gamma(r) = \int_0^{\infty} x^{r-1} e^{-x}\, dx$$

*From 1 we substitute Expected value*

$$(x) = \int_0^{\infty} \mathrm{x f}(x)\, dx =$$
$$= \int_0^{\infty} x \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}\, dx$$
$$= \int_0^{\infty} \frac{\lambda^r}{\Gamma(r)} x^{r-1+1} e^{-\lambda x}\, dx$$
$$= \frac{\lambda^r}{\Gamma(r)} \int_0^{\infty} x^{r-1+1} e^{-\lambda x}\, dx$$

*Making variable change:*

$$y = \lambda x \Rightarrow x = \frac{y}{\lambda} \Rightarrow dx = \frac{dy}{\lambda}$$
$$\Rightarrow \frac{\lambda^r}{\Gamma(r)} \int_0^{\infty} x^{r-1+1} e^{-\lambda x}\, dx$$
$$= \frac{\lambda^r}{\Gamma(r)} \int_0^{\infty} \left(\frac{y}{\lambda}\right) e^{-y r} \frac{dy}{\lambda}$$
$$= \frac{\lambda^r}{\Gamma(r)} \int_0^{\infty} \frac{y^r}{\lambda^r} e^{-y} \frac{dy}{\lambda}$$
$$= \frac{1}{\lambda\Gamma(r)} \int_0^{\infty} y^r e^{-y}\, dy$$

*Observation:*

$$\int_0^{\infty} y^r e^{-y}\, dy = \int_0^{\infty} y^{r+1-1} e^{-y}\, dy$$

*looks like a Gamma function with r+1*

$$\int_0^{\infty} y^r e^{-y}\, dy = \int_0^{\infty} y^{r+1-1} e^{-y}\, dy =$$
$$= \Gamma(r+1)$$

$$\text{If:} \quad \frac{1}{\lambda\Gamma(r)} \int_0^{\infty} y^r e^{-y}\, dy$$
$$= \frac{1}{\lambda\Gamma(r)} \int_0^{\infty} y^{r+1-1} e^{-y}\, dy$$
$$= \frac{1}{\lambda\Gamma(r)} \int_0^{\infty} y^r e^{-y}\, dy$$
$$= \frac{\Gamma(r+1)}{\lambda\Gamma(r)}$$

*Applying Gamma Properties:* $\Gamma(r+1) = (r+1-1)\Gamma(r+1-1) = r\,\Gamma(r)$

$$\Rightarrow \frac{\Gamma(r+1)}{\lambda\Gamma(r)} = \frac{r\Gamma(r)}{\lambda\Gamma(r)} = \frac{r}{\lambda}$$

**Thus:** $\boldsymbol{E(x) = \frac{r}{\lambda}}$

*From equation 3: Var(x) = E(x²) - (E(x))²*

$$\text{Let:} E(x^2) = \int_0^{\infty} x^2 f(x)\, dx$$
$$= \int_0^{\infty} x^2 \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}\, dx =$$
$$\int_0^{\infty} \frac{\lambda^r}{\Gamma(r)} x^{r+2-1} e^{-\lambda x}\, dx =$$
$$= \frac{\lambda^r}{\Gamma(r)} \int_0^{\infty} \frac{\lambda^r}{\Gamma(r)} x^{r+2-1} e^{-\lambda x}\, dx$$

$$\textit{Changing the variable}:$$
$$y = \lambda x \Rightarrow x = \frac{y}{\lambda} \Rightarrow dx = \frac{dy}{\lambda}$$
$$\Rightarrow \frac{\lambda^r}{\Gamma(r)} \int_0^{\infty} x^{r+2-1} e^{-\lambda x}\, dx$$
$$= \frac{\lambda^r}{\Gamma(r)} \int_0^{\infty} \left(\frac{y}{\lambda}\right)^{r+2-1} e^{-y} \frac{dy}{\lambda} =$$
$$= \frac{\lambda^r}{\lambda\Gamma(r)} \int_0^{\infty} \frac{y^{r+2-1}}{\lambda^{r+2-1}} e^{-y}\, dy$$
$$= \frac{\lambda^r}{\lambda\Gamma(r)} \int_0^{\infty} \frac{y^{r+2-1}}{\lambda^{r+1}} e^{-y}\, dy$$
$$= \frac{\lambda^r}{\lambda\lambda^{r+1}\Gamma(r)} \int_0^{\infty} y^{r+2-1} e^{-y}\, dy$$
$$= \frac{\lambda^r}{\lambda^2\lambda^r\Gamma(r)} \int_0^{\infty} y^{r+2-1} e^{-y}\, dy$$
$$= \frac{1}{\lambda^2\Gamma(r)} \int_0^{\infty} y^{r+2-1} e^{-y}\, dy =$$

$$\textit{Observation}:$$
$$\int_0^{\infty} y^{r+2-1} e^{-y}\, dy = \Gamma(r+2)$$
$$\Rightarrow \frac{1}{\lambda^2\,\Gamma(r)} \int_0^{\infty} y^{r+2-1} e^{-y}\, dy$$
$$= \frac{\Gamma(r+2)}{\lambda^2\Gamma(r)}$$

*Applying Gamma Properties:*

Γ(r+2) = (r+2-1)Γ(r+2-1) = (r+1) Γ(r+1) = (r+1) (r+1-1) Γ(r+1-1) = r(r+1)Γ(r)

$$\Rightarrow \frac{\Gamma(r+2)}{\lambda^2\,\Gamma(r)} =$$

$$\frac{r(r+1)\Gamma(r)}{\lambda^2\Gamma(r)} = \frac{r(r+1)}{\lambda^2}$$

$$Thus: E(x^2) = \frac{r(r+1)}{\lambda^2}$$

$$\mathrm{Var}(x) = \mathrm{E}(x^2) - (E(x))^2$$

$$= \frac{r(r+1)}{\lambda^2} - (\frac{r^2}{\lambda^2}) =$$

$$\frac{\mathrm{r}(r+1)}{\lambda^2} - \frac{r^2}{\lambda^2} = \frac{\mathrm{r}(r+1) - r^2}{\lambda^2}$$

$$= \frac{r^2 + r - r^2}{\lambda^2} = \frac{r}{\lambda^2}$$

$$\textbf{Thus}: \mathbf{Var}(\boldsymbol{x}) = \frac{\boldsymbol{r}}{\boldsymbol{\lambda^2}}$$

A random variable Y is said to have a beta probability distribution with parameters α and β if and only if the density function of X is

$$f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}, \mathrm{I}_{(0,1)}(x)$$

with: α > 0 y β > 0;

$$Where: B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx$$

$$Properties: B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

Γ(α) = (α-1) Γ(α-1) and Γ(α) = (α-1)! *when α is an integer From 1 we substitute Expected value*

$$E(x) = \int_0^1 \mathrm{x f}(x)\,dx$$

$$= \int_0^1 x \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}\,dx$$

$$\Rightarrow \frac{1}{B(\alpha,\beta)} \int_0^1 \mathrm{x x}^{\alpha-1}(1-x)^{\beta-1}\,dx$$

$$= \frac{1}{B(\alpha,\beta)} \int_0^1 x^{\alpha-1+1}(1-x)^{\beta-1}\,dx =$$

$$= \frac{1}{B(\alpha,\beta)} \int_0^1 x^{(\alpha+1)-1}(1-x)^{\beta-1}\,dx$$

$$\Rightarrow \frac{B(\alpha+1,\beta)}{B(\alpha,\beta)} = \frac{\Gamma(\alpha+1)\Gamma(\beta)}{\Gamma(\alpha+1+\beta)} \Big/ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} =$$

$$= \frac{\Gamma(\alpha+1)\,\Gamma(\beta)\,\Gamma(\alpha+\beta)}{\Gamma(\alpha+1+\beta)\,\Gamma(\alpha)\,\Gamma(\beta)}$$

$$= \frac{\Gamma(\alpha+1)\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+1)\Gamma(\alpha)}$$

*Making:*
Γ(α+1) = (α+1-1)Γ(α+1-1) = αΓ(α)
Γ(α+β+1) = (α+β+1-1)Γ(α+β+1-1)
= (α+β)Γ(α+β)

$$\Rightarrow \frac{\Gamma(\alpha+1)\,\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+1)\,\Gamma(\alpha)}$$

$$= \frac{\alpha\Gamma(\alpha)\Gamma(\alpha+\beta)}{(\alpha+\beta)\Gamma(\alpha+\beta)\Gamma(\alpha)} = \frac{\alpha}{(\alpha+\beta)}$$

$$\textbf{Thus}: \boldsymbol{E}(\boldsymbol{x}) = \frac{\boldsymbol{\alpha}}{(\boldsymbol{\alpha}+\boldsymbol{\beta})}$$

*From 3 we substitute Variance*
*Var(x) = E($x^2$) - (E(x))$^2$*

$$E(x^2) = \int_0^1 x^2\, \mathrm{f}(x)\,dx$$

$$= \int_0^1 x^2 \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}\,dx$$

$$\Rightarrow \frac{1}{B(\alpha,\beta)} \int_0^1 x^2 x^{\alpha-1}(1-x)^{\beta-1}\,dx$$

$$= \frac{1}{B(\alpha,\beta)} \int_0^1 x^{\alpha-1+2}(1-x)^{\beta-1}\,dx =$$

$$= \frac{1}{B(\alpha,\beta)} \int_0^1 x^{(\alpha+2)-1}(1-x)^{\beta-1}\,dx$$

$$\Rightarrow \frac{B(\alpha+2,\beta)}{B(\alpha,\beta)} = \frac{\Gamma(\alpha+2)\Gamma(\beta)}{\Gamma(\alpha+2+\beta)} \Big/ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} =$$

$$= \frac{\Gamma(\alpha+2)\,\Gamma(\beta)\,\Gamma(\alpha+\beta)}{\Gamma(\alpha+2+\beta)\,\Gamma(\alpha)\,\Gamma(\beta)}$$

$$= \frac{\Gamma(\alpha+2)\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+2)\Gamma(\alpha)}$$

*Making: Γ(α+2) = (α+2-1)Γ(α+2-1)*
*= (α+1)Γ(α+1)*
*Γ(α+β+2) = (α+β+2-1)Γ(α+β+2-1)*
*= (α+β+1)Γ(α+β+1)*
*Γ(α+1) = (α+1-1)Γ(α+1-1) = αΓ(α)*
*Γ(α+β+1) = (α+β+1-1)Γ(α+β+1-1)*
*= (α+β)Γ(α+β)*

$$\Rightarrow \frac{\Gamma(\alpha+2)\Gamma(\alpha+\beta)}{\Gamma(\alpha+\beta+2)\Gamma(\alpha)}$$

$$= \frac{(\alpha+1)\Gamma(\alpha+1)\Gamma(\alpha+\beta)}{(\alpha+\beta+1)\Gamma(\alpha+\beta+1)\Gamma(\alpha)} =$$

$$= \frac{(\alpha+1)\alpha\Gamma(\alpha)\Gamma(\alpha+\beta)}{(\alpha+\beta+1)(\alpha+\beta)\Gamma(\alpha+\beta)\Gamma(\alpha)}$$

$$= \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}$$

$$\text{Thus } E\,(x^2) =$$

$$= \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)}$$

*Si Var(x) = E($x^2$) - (E(x))$^2$*

$$= \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)} - (\frac{\alpha}{(\alpha+\beta)})^2 =$$

$$= \frac{\alpha(\alpha+1)}{(\alpha+\beta)(\alpha+\beta+1)} - \frac{\alpha^2}{(\alpha+\beta)^2}$$
$$= \frac{\alpha(\alpha+1)(\alpha+\beta)^2 - \alpha^2(\alpha+\beta)(\alpha+\beta+1)}{(\alpha+\beta)(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$= \frac{\alpha^3+\alpha^2\beta+\alpha^2+\alpha\beta-\alpha^3-\alpha^2\beta-\alpha^2}{(\alpha+\beta)^2(\alpha+\beta+1)} =$$
$$= \frac{\alpha^3+\alpha^2\beta+\alpha^2+\alpha\beta-\alpha^3-\alpha^2\beta-\alpha^2}{(\alpha+\beta)^2(\alpha+\beta+1)}$$
$$= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$
$$\textbf{Thus Var}(\boldsymbol{x}) =$$
$$= \frac{\boldsymbol{\alpha\beta}}{(\boldsymbol{\alpha}+\boldsymbol{\beta})^2(\boldsymbol{\alpha}+\boldsymbol{\beta}+\mathbf{1})}$$

**Problems**

1. If a skydiver lands at a random point on a straight line between markers A and B, find the probability that she is closer to A than to B. Find the probability that her distance to A is more than three times her distance to B.

2. A phone call arrives at a random switch within a minute. The switch was fully occupied for 15 seconds within this one-minute period. What is the probability that the call will arrive when the switch has not been fully occupied?

3. According to Zimmels (1983), the particle sizes used in sedimentation experiments often have a uniform distribution. In sedimentation involving mixtures of particles of various sizes, the larger ones prevent the movement of the smaller ones. Therefore, it is important to study the mean and variance of particle sizes. Assume that spherical particles have diameters that are uniformly distributed between .01 and .05 centimeters. Find the mean and variance of the volumes of these particles. (Remember that the volume of a sphere is (4/3)\[Pi] r^3)

4. It was noted that the weekly amount of money spent by a company over a long period of time on maintenance and repairs is normally distributed approximately with an average of $400 and a standard deviation of $20. If $450 is budgeted for the next week, what is the probability that the actual costs will exceed the budgeted amount?

5. It is specified that cables manufactured for use in a computer system must have resistances between .12 and .14 ohms. The actual measured resistances of the cables produced by Company A have a normal probability distribution with a mean of .13 ohms and a standard deviation of .005 ohm.
   a. What is the probability that a cable selected at random from Company A's production will meet the specifications?

6. Assume that a random variable X has a Beta distribution with \[Alpha]= 1 and \[Beta]= 3,
   (a) Determine the mean and median of X

7. If the proportion of a TV brand that requires service during the first year of operation is a random variable that has a Beta distribution with 'Alpha' = 3 and 'Beta' = 2, what is the probability that at least 80% of the new models of this brand sold this year will require service during their first year of operation?

8. In a certain city, daily water consumption (millions of liters) follows approximately a Gamma distribution with r=2 and \[Lambda]=1/3. If the daily capacity of that city is 9 million liters of water, what is the probability that on any given day the water supply will be inadequate?

9. Assume that the time, in hours, needed to repair a heat pump is a random variable X that has a Gamma distribution with the parameters r=2 and \[Lambda]=2. What is the probability that the next call will require:
   a. at most one hour to repair the pump?

10. In a biomedical investigation, it was determined that the survival time, in weeks, of an animal when subjected to certain exposure to Gamma radiation, has a Gamma distribution with r=5 and \[Lambda]=1/10.

a. What is the average survival time of a randomly selected animal of the type used in the experiment?

**Conclusions**

In this work, concepts and proofs have been presented using clear and detailed language sufficient for a higher-level mathematician, although in other cases not necessarily for one at the level of an engineering area, a biologist, ecologist, or sociologist using statistics. In order for the student to understand the meanings and procedures performed to achieve the proposed objective, this includes the use of the algebraic artifices supported by differential and integral calculus.

Other essential aspects are:

a. That the student acquires mastery and understanding of the basic concepts of Axiomatic Probability.
b. That the students develop the autonomous character to build and develop logical arguments under proofs.
c. Students are expected to express themselves correctly using the formal language of Mathematics from knowledge already acquired in this work and previous ones.

**Suggestions (future work)**

To further enrich this work, we suggest developing proofs using the moment-generating functions as another way to obtain the expressions of expected value and variance of the main probability distributions.

Perform simulations using mathematical software such as R, Matlab, Mathematica, to name a few, simulating the behavior of major probability distributions with examples from real applications.

We suggest applying this approach to groups of higher-level students in order to assess the scope of the approach.

Develop proofs of the mode and median of these distributions presented in this paper.

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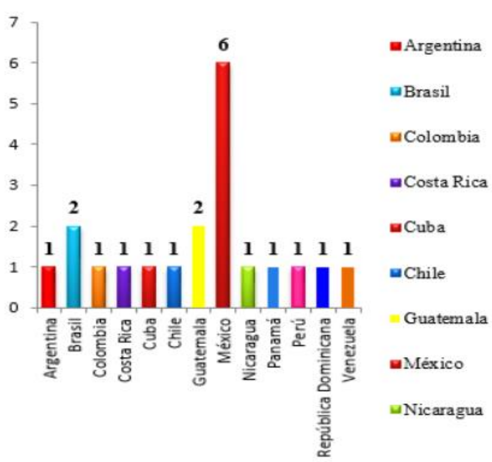


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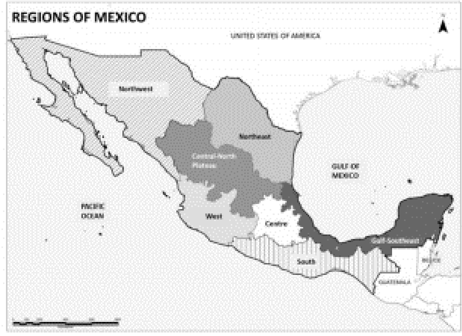


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